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Parameter identification of low-order boiler models for dynamic stability analysis

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I. INTRODUCTION

In recent years there has been growing concern about the effect on the dynamic stability of electric power systems of major power changes. Some possible major changes are a fault in the network, failure of a piece of equipment such as a generating unit, or sudden application of a major load. The slowest but most important type of transients following a major disturbance in power systems are electromechanical generator rotor swings. The electromechanical transients are characterized by an electromechanical torque and power balance within each machine. As a result of disturbances, the torque balance is upset within each machine. The individual machines will be subject to accelerations or decelerations, causing angular rotor swings of such large magnitudes that certain machines may pull out of synchronism.

During approximately the first second following a disturbance, the turbine torque to the generator shaft and the electromagnetic force behind the transient reactance remain fairly constant. In this time interval, transient behavior can be adequately described by a relatively simple representation of each generator and of each prime mover. Additional detail, especially on the prime mover (boiler, turbine and generator) dynamics must be added in order to carry the simulation further in time. The slower reacting components in a power system may be neglected if only a short period (a few seconds) is being simulated, but they could be very important and should not be neglected. Inclusion of the boiler is usually considered necessary where simulations extend beyond 10 to 20 seconds (1).

Stability analysis is usually done by means of a computer simulation. It is important to have an accurate mathematical model of the physical system being simulated. Also, since power systems are generally large and complex, it is necessary to simplify models of individual components of the system as much as possible.

This dissertation is primarily concerned with obtaining a set of parameters needed for a model of the boiler which is felt to reasonably describe the dynamic behavior of the system while being of feasible size.

Process models can be obtained from basic physical laws, from pure input/output measurements, or from a combination of these. The modeling from physical principles is widely used and has the advantage of a wide range of validity. Some drawbacks of physical modeling are that the procedure is time-consuming, it leads to very complex and large scale models, and the model must ultimately be compared with actual operating data anyway. Our approach is to use the input/output measurement approach in combination with some physical considerations.

The case of pure input/output modeling consists of determining a model from input/output measurements only. This is often called the black box approach. The advantage of this method is that it is usually done quickly, and it often leads to fairly simple, low-order models.

Low-order models are usually accurate only over a short interval of time following a disturbance from normal operating conditions and may not give accurate results for simulations of several minutes

duration. However, the models can be useful in power system stability studies.

A number of low-order boiler models have been proposed for steam power plants dynamic study. Many references are listed and reviewed in recent papers (1, 2). A few attempts have been made to identify the parameters of the low-order model with a particular plant process such as drum or once-through boilers. Speedy, et al. (3), have attempted to identify the parameters of the low-order boiler model by using differential approximation, a least squares fitting between the derivatives of the variables measured on the plant and those computed from the functions on the right hand side of the model equation. The algorithm is non-iterative, using only one iteration to obtain a solution. It was intended that these approximate parameter values would be used as initial values in some more sophisticated scheme for refining the estimates. Extensive study on the parameter identification of boiler models has been done by Eklund and Gustavsson (4). Most results are based on single input experiments where one input at a time is perturbed.

It is more desirable to have a model where the identification of parameters is based on multiple input experiments where all inputs are purturbed simultaneously. The boiler-turbine unit is a typical multiple-input/multiple-output system. The identification problem is considerably more difficult to handle when the process is a multiple-input/multiple output system. Some of the difficulties are: (1) the

ways of test procedure, (2) choice of model structure and unknown parameters, and (3) enormous computational time.

This dissertation presents some parameter identification results of multiple-input/multiple-output low-order boiler models based on field test data which were obtained by purturbing all inputs to the boiler simultaneously.

This dissertation is organized as follows: Chapter 1 is an introduction to the dissertation. It defines the problem to be investigated, describes the background and objectives of the research, and summarizes the contents of each chapter. Chapter 2 formulates the identification problem and develops iterated linear regression analysis as a technique for the parameter identification of a dynamic system. Several other possible methods are discussed. Chapter 3 investigates the properties of the iterative regression analysis method. Chapter 4 examines the parameter identifiability of a linear dynamic system. Chapter 5 reviews boiler process and control and examines several proposed low-order boiler models. Chapter 6 develops a computational procedure for the application of a least squares regression method to the parameter identification of a proposed low-order boiler model under simulated conditions. It examines the convergence properties of the algorithm. Chapter 7 describes field test procedures of boiler dynamics and defines each test. Chapter 8 shows the results of the parameter identification of a low-order model from the actual field test data. Chapter 9 summarizes the results of the research.

II. LEAST SQUARES PARAMETER ESTIMATES

A. Formulation of Problem

The general model to be identified is represented by the following vector differential equations.

$$\dot{\underline{x}} = \underline{g}_x (\underline{x}, \underline{p}, t), \ \underline{x}(0) = \underline{x}_o \tag{2.1}$$

where: $\underline{x}$ is an m component state vector,

$\underline{g}$ is a known m component vector function,

$\underline{p}$ is a vector of unknown stationary parameters,

and $\underline{x}_o$ is an m vector of initial conditions on $\underline{x}$ which may or may not be completely known.

Since $\underline{p}$ is stationary, $\dot{\underline{p}} = 0$. Thus, $\underline{p}$ can be adjoined to $\underline{x}$ to give a new n(=m+s) component state vector $\underline{z}$.

$$\text{Let } \underline{z} = \begin{bmatrix} \underline{x} \\ \underline{p} \end{bmatrix} . \tag{2.2}$$

$$\text{Then } \dot{\underline{z}} = \begin{bmatrix} \dot{\underline{x}} \\ \cdot \\ \underline{p} \end{bmatrix} = \begin{bmatrix} \underline{g}_x (\underline{x}, \underline{p}, t) \\ 0 \end{bmatrix} = \underline{g}_z (\underline{z}, t) \tag{2.3}$$

with the initial conditions

$$\underline{z}(0) = \begin{bmatrix} \underline{x}(0) \\ \underline{p} \end{bmatrix} = \begin{bmatrix} \underline{x}_o \\ \underline{p} \end{bmatrix} .$$

Let the measured output vector, $\underline{Y}(N)$, and the model output vector $\underline{f}(\underline{z}(0))$ be defined as:

$$\underline{Y}(N) = \begin{bmatrix} \underline{y}_m(t_o) \\ \underline{y}_m(t_1) \\ \cdot \\ \cdot \\ \cdot \\ \underline{y}_m(t_N) \end{bmatrix}, \text{ and } \underline{f}(\underline{z}(0)) = \begin{bmatrix} \underline{h}(\underline{z}(t_o), t_o) \\ \underline{h}(\underline{z}(t_1), t_1) \\ \cdot \\ \cdot \\ \cdot \\ \underline{h}(\underline{z}(t_N), t_N) \end{bmatrix} \quad (2.4) \quad (2.5)$$

where $\underline{y}_m(t_i)$ and $\underline{h}(\underline{z}(t_i), t_i)$ are the measured output and the model output respectively, sampled at $t = t_i$. The model output, $\underline{h}(\underline{z}(t_i), t_i)$, is sometimes denoted as $\underline{y}(t_i)$.

A residual vector $\underline{\epsilon}$ is defined as:

$$\underline{\epsilon} = \begin{bmatrix} \underline{\epsilon}_o \\ \cdot \\ \cdot \\ \cdot \\ \underline{\epsilon}_N \end{bmatrix} \quad (2.6)$$

where $\underline{\epsilon}_i = \underline{y}_m(t_i) - \underline{h}(\underline{z}(t_i), t_i)$.

Based on the observations, $\underline{Y}(N)$, we want to find $\underline{z}(0)$ such that we have a least squared error fit with the data. That is, the performance index

$$J(\underline{z}(0)) = \left[\underline{Y}(N) - \underline{f}(z(0))\right]^T W \left[\underline{Y}(N) - \underline{f}(z(0))\right] \quad (2.7)$$

is minimized subject to the constraint

$$\underline{z}(t) = \underline{g}_z(\underline{z}(t), t), \qquad \underline{z}(0) = \underline{z}_o. \quad (2.8)$$

The weighting matrix W may be any positive definite matrix, and T denotes the transpose of a matrix.

B. Iterated Linear Regression Method

We assume that an initial estimate $\underline{\hat{z}}^i(0)$ is known. We wish to obtain a new estimate $\underline{\hat{z}}^{i+1}(0)$ which more nearly minimizes the J of Equation 2.7. Assuming that $\underline{z}^{i+1}(0)$ is sufficiently close to some known $\underline{\hat{z}}^i(0)$, $\underline{f}(\underline{z}^{i+1}(0))$ can be expanded in a Taylor series about $\underline{\hat{z}}^i(0)$ as

$$\underline{f}(\underline{z}^{i+1}(0)) \cong \underline{f}(\underline{\hat{z}}^i(0)) + F(\underline{\hat{z}}^i(0)) \left[\underline{z}^{i+1}(0) - \underline{\hat{z}}^i(0) \right] \qquad (2.9)$$

where $$F(\hat{\underline{z}}\ (0)) = \left. \frac{\partial \underline{f}(z(0))}{\partial \underline{z}(0)} \right|_{\underline{z}(0) = \underline{\hat{z}}^i(0)}$$

i.e. $F(\underline{z}(0))$ is the Jacobian matrix whose ij-th element $\left[\partial \underline{f} / \partial \underline{z}(0) \right]_{ij}$ is the partial derivative of the i-th component of $\underline{f}(\underline{z}(0))$ with respect to the j-th component of $\underline{z}(0)$.

If we substitute $\underline{f}(\underline{z}(0))$ from Equation 2.9 into J, we obtain the following linearized performance index:

$$J_L\ (\underline{z}^{i+1}(0)) = \left[\underline{Y}(N) - \underline{f}(\underline{z}^i(0)) - F(\underline{z}^i(0)) \left[\underline{z}^{i+1}(0) - \underline{z}^i(0) \right] \right]^T W$$
$$\left[\underline{Y}(N) - \underline{f}(\underline{z}^i(0)) - F(\underline{z}^i(0)) \left[z^{i+1}(0) - z^i(0) \right] \right] \qquad (2.10)$$

By taking the partial derivative of J_L with respect to $\underline{z}^{i+1}(0)$ and setting the result equal to zero, we find that

$$\underline{\hat{z}}^{i+1}(0) = \left[F^T(\underline{\hat{z}}^i(0)) W\, F(\underline{\hat{z}}^i(0)) \right]^{-1} F^T(\underline{\hat{z}}^i(0))\ W \left[\underline{Y}(N) - \underline{f}(\underline{\hat{z}}^i(0)) \right] + \underline{\hat{z}}^i(0) \qquad (2.11)$$

where

$$F(\hat{\underline{z}}^i(0)) = \left.\frac{\partial \underline{f}(\underline{z}(0))}{\partial \underline{z}(0)}\right|_{\underline{z}(0) = \hat{\underline{z}}^i(0)}$$

provided that $\left[F^T(\hat{\underline{z}}^i(0)\ W\ F(\hat{\underline{z}}^i(0))\right]^{-1}$ exists. (2.12)

With $\underline{z}^{i+1}(0)$ known, we may now determine the entire trial trajectory $\underline{z}^{i+1}(t)$ and compute $F(\underline{z}^{i+1}(0))$. We repeat the process until the solution changes by only a small amount from iteration to iteration. When the difference between two iterations is less than some predetermined value, it will be assumed that it is sufficiently close to the desired value that minimizes the performance index J.

The iterative method given in Equation 2.11 is called Gauss-Newton iterative method (5). We are going to review some other possible methods in the next section.

C. Gradient Methods

Much of the following discussion is based on (6-8). The method of minimizing the performance index, J(z(0)), Equation 2.7 is to compute a root of the equation

$$\nabla J\ (\underline{z}(0)) \triangleq \frac{\partial J\ (\underline{z}(0))}{\partial \underline{z}(0)} = 0 \qquad (2.13)$$

by some iterative procedure such that

$$\underline{z}^{i+1}(0) = \underline{z}^i(0) + K^i\ \nabla J(\underline{z}^i(0)) \qquad i = 0, 1, 2 \ldots \qquad (2.14)$$

The problem to be solved next is the determination of suitable matrices K^i at each iteration, i. Clearly, if

$$K^i = -\tfrac{1}{2}\left[F^T(\underline{z}^i(0))\ W\ F\ (\underline{\hat{z}}^i(0))\right]^{-1} \qquad (2.15)$$

then the algorithm is the Gauss-Newton iteration scheme.

A summary of various proposed iteration schemes (6, 7, 8, 9) is presented in the Table 2.1.

Table 2.1. Iteration methods

Name	K^i
Steepest Desent	$K^i = -kI$
Newton-Raphson	$K^i = \dfrac{-J(\underline{z}^i(0))}{\|\nabla J(\underline{z}^i(0))\|^2}$
Newton	$K^i = -\left[\dfrac{\partial J(\underline{z}^i(0))}{\partial \underline{z}^i(0)\ \partial \underline{z}^i(0)}\right]^{-1}$
Gauss-Newton	$K^i = -\tfrac{1}{2}\left[F^T(z^i(0))\ W\ F\ (z^i(0))\right]^{-1}$

The algorithms shown in Table 2.1 may converge or may fail to converge depending upon initial parameters, data and system structures of interest. Some modified versions of the above algorithms are the modified Gauss-Newton Method (10) and the Fletcher and Powell Method (9).

The classical steepest descent method is based on adjusting parameters along a line parallel to the local gradient vector, i.e.

$$\Delta \underline{z}^i(0) = K \nabla J\ (\underline{z}^i(0)) \qquad (2.16)$$

and then choosing the next parameter values as

$$\underline{z}^{i+1}(0) = \underline{z}^{i}(0) + \Delta \underline{z}^{i}(0) \tag{2.17}$$

In Equation 2.16, the quantity K is a constant matrix

$$K = -kI, \quad k > 0$$

where k is a scalar constant and I is the identity matrix.

The new value $\underline{z}^{i+1}(0)$ in turn, provides a basis for the determination of a new gradient vector for the next cycle of iteration. When this is done, it seems reasonable that universally stable algorithms for the minimization of sum-squared error may be developed since it will be possible to test a tentative new value for the parameter vector before accepting it as a better approximation to the true parameter vector. Usually the initial guess of parameter values is not crucial to convergence but the algorithm approaches a minimum rapidly, then slows down drastically. Since each component of the gradient vector approaches zero as the minimum is approached, the step size also approaches zero. Hence, the steepest descent method has poor convergence properties near the minimum of quadratic criterion functions. Methods which overcome this difficulty will be discussed further.

The Newton-Raphson method (6) is based on linear extrapolation of J to zero using the first two terms of a Taylor's series expansion.

We can write

$$J(\underline{z}(0)) \cong J(\underline{z}^i(0)) + \nabla J(\underline{z}^i(0))(\underline{z}(0) - \underline{z}^i(0)) \qquad (2.18)$$

If we select the value of $\underline{z}(0) = \underline{z}^{i+1}(0)$ to be the one for which $\nabla J(\underline{z}(0)) = 0$ we obtain

$$\Delta \underline{z}^i(0) = K \nabla J(\underline{z}^i(0)) \qquad (2.19)$$

where $K^i = \dfrac{-J(\underline{z}^i(0))}{\nabla J(\underline{z}^i(0))^T \nabla J(\underline{z}^i(0))}$

While the value obtained from equation 2.19 may work quite well in the early phases of an iterative parameter scheme, it is not suitable for the terminal stages of iteration unless the performance index, J, attains the value zero at its minimum. It produces excessively large steps near the minimum. This can be seen from a consideration of the limiting behavior of $\Delta \underline{z}^i(0)$:

$$\lim_{i \to \infty} \left| \Delta z^i(0) \right| = \lim_{\nabla J(\underline{z}(0)) \to 0} \frac{J(\underline{z}(0))}{\left| \nabla J(\underline{z}(0)) \right|} = \infty \qquad (2.20)$$

unless $J(\underline{z}(0)) \to 0$. Since the function, J, does not usually attain the value zero when real physical data is involved, another way of determining the suitable matrices K must be provided.

Newton's method of iteration (11) is also based on gradient information. Near the minimum, the performance index can be represented by the first three terms of the Taylor Series expansion around the current estimate of the parameter $\underline{z}^i(0)$:

$$J(\underline{z}(0)) \cong J(\underline{z}^i(0)) + \nabla J(\underline{z}^i(0))^T \, (\underline{z}(0) - \underline{z}^i(0))$$

$$+ \tfrac{1}{2} \, (\underline{z}(0) - \underline{z}^i(0))^T \, H \, (\underline{z}(0) - \underline{z}^i(0)) \qquad (2.21)$$

where H is the Hessian matrix of J and is defined as

$$H(\underline{z}^i(0)) \triangleq \frac{\partial \nabla J(\underline{z}(0))}{\partial \underline{z}(0)} \Bigg|_{\underline{z}(0) = \underline{z}^i(0)} \qquad (2.22)$$

To find the value of $\underline{z}^{i+1}(0)$ which minimizes (2.21), we differentiate the equation and set the derivative equal to zero.

$$\nabla J(\underline{z}^{i+1}(0)) = 0 = \nabla J(\underline{z}^i(0)) + H(\underline{z}^i(0)) \, (\underline{z}^{i+1}(0) - \underline{z}^i(0)) \qquad (2.23)$$

We have

$$\Delta \underline{z}^i(0) = \underline{z}^{i+1}(0) - \underline{z}^i(0) = K^i \nabla J(\underline{z}^i(0)) \qquad (2.24)$$

where $K^i = -\left[H(\underline{z}^i(0)) \right]^{-1}$

This method has a strong convergence property known as "quadratic convergence" which simply means that if J is in fact a quadratic function of $\underline{z}^i(0)$, the algorithm converges in a single step. This method is appealing because of its convergence properties, but difficult to implement because it is necessary to compute the Hessian matrix $H(\underline{z}^i(0))$, at each iteration, i. The method described by Fletcher and Powell (9) retains the quadratic convergence properties of Newton's Method near the minimum of the function, J, and circumvents the difficulties involved in computing the inverse of the Hessian matrix. However the method is complicated to implement and to be

analyzed from the viewpoint of dynamic system structure, e.g., system controllability and observability compared to the Gauss-Newton method.

The Gauss-Newton scheme is based on the fact that linear estimation techniques can be applied to the estimation of $\underline{z}(0)$ in Equation 2.5 if the non-linear function $\underline{f}(\underline{z}(0))$ is represented by the linear relationship of Equation 2.9. If we compare Equations 2.11 and 2.16, then we can find a similarity between the steepest descent method and the Gauss-Newton method. The only difference is that in this method the projection is rotated from the gradient direction by the transformation matrix,

$$K^i = -\tfrac{1}{2}\left[F\,(\underline{z}(0))\; W\; F\,(\underline{z}\,(0))\right]^{-1} .$$

Thus the Gauss-Newton iteration is based on a linearly transformed step along the gradient. This transformation produces both a magnitude and an angle change in $\nabla J(\underline{z}(0))$ and may well fail to produce a convergent estimate of the true parameter vector when there is an angle change of more than 90°.

The Modified Gauss-Newton Method takes this into account by inserting a scale factor ν ($0 \leq \nu \leq 1$) into the basic iteration equation:

$$\underline{z}^{i+1}(0) = \underline{z}^{i}(0) + \nu^{i}\ \Delta\underline{z}^{i}(0) \qquad (2.25)$$

Hartley (10) has proven there always exists a scale factor for which Equation 2.25 will produce a convergent iteration. In addition, he has developed a computational procedure based on searching

$J(\underline{z}^{i+1}(0))$ at each iteration for the minimum with respect to ν and then using this value of ν to continue on to the next iteration.

The Gauss-Newton method is basically implemented throughout this dissertation. The algorithm is easier to visualize and also faster to mechanize computationally than other methods described in this section. When there is no information for a good initial guess of parameter values, the Newton-Raphson method is utilized. The modified Gauss-Newton algorithm is sometimes used when the Gauss-Newton algorithm fails to converge to the assumed true parameter values. The computational procedures implementing the above computation policy is presented later in Chapter 6.

In summary, the parameter identification problem of dynamic systems is formulated in least squares regression analysis, and several possible iterative methods for the automatic determination of parameter vectors have been investigated in this chapter.

III. PROPERTIES OF ITERATIVE LINEAR REGRESSION METHOD

A. Jacobian Matrix Calculation of Linear System

From Equation 2.9, the Jacobian matrix is defined as

$$F(\underline{z}(0)) \triangleq \frac{\partial \underline{f}(\underline{z}(0))}{\partial \underline{z}(0)}$$

$$= \begin{bmatrix} \left(\frac{\partial \underline{h}(\underline{z}(0))}{\partial \underline{z}(0)}\right) \\ \left(\frac{\partial \underline{h}(\underline{z}(t))}{\partial \underline{z}(t)}\right) \cdot \left(\frac{\partial \underline{z}(t)}{\partial \underline{z}(0)}\right) \Bigg| \; t = t_1 \\ \vdots \\ \left(\frac{\partial \underline{h}(\underline{z}(t))}{\partial \underline{z}(t)}\right) \left(\frac{\partial \underline{z}(t)}{\partial \underline{z}(0)}\right) \Bigg| \; t = t_N \end{bmatrix} \tag{3.1}$$

If we define the matrix of state sensitivity functions as

$$\Phi(t) \triangleq \frac{\partial \underline{z}(t)}{\partial \underline{z}(0)} \tag{3.2}$$

then the matrix has the following property

$$\dot{\Phi}(t) = \frac{\partial \underline{g}_z(\underline{z}(t),t)}{\partial \underline{z}(t)} \cdot \Phi(t) \tag{3.3}$$

with $\Phi(0) = I$.

The proof of this statement is seen as follows.

$$\frac{d}{dt}\,\Phi(t) = \frac{d}{dt}\,\frac{\partial\,\underline{z}(t)}{\partial\,\underline{z}(0)} = \frac{\partial}{\partial\,z(0)} \cdot \frac{d}{dt}\,\underline{z}(t) = \frac{\partial}{\partial\,\underline{z}(0)}\;\dot{z}(t)$$

$$= \frac{\partial}{\partial\,\underline{z}(0)}\,\underline{g}_z\;(\underline{z}(t),\,t)$$

$$= \frac{\partial}{\underline{z}(t)}\,\underline{g}_z\;(\underline{z}(t),\,t) \cdot \frac{\partial}{\partial\,\underline{z}(0)}\,\underline{z}(t)$$

$$= \frac{\partial}{\partial\,\underline{z}(t)}\,\underline{g}_z\;(\underline{z}(t),\,t) \cdot \Phi(t)$$

Therefore $\dfrac{d}{dt}\,\Phi(t) = \dfrac{\partial}{\partial\,\underline{z}(t)}\,\underline{g}_z\;(\underline{z}(t),\,t) \cdot \Phi(t)$

and $\Phi(0) = \dfrac{\partial\,\underline{z}(0)}{\partial\,\underline{z}(0)} = I$

It has been shown that the computation of the sensitivity can be reduced for the time-invariant linear systems by Wilkie and Perkins (12) and Denery (13) when the initial state vector $\underline{z}(0)$ is known. Usually in process identification, the initial vector is unknown. If we assume that the system to be identified is linear, then Equation 2.1 can be written as

$$\dot{\underline{x}} = \underline{g}_x\;(\underline{x},\,\underline{p},\,t) = A\;(\underline{p})\;\underline{x} + B\;(\underline{p})\;\underline{u} \qquad (3.4)$$

with $\underline{x}(0) = \underline{x}_0$

where: $A(\underline{p})$ is an n x n matrix (elements are unknown)

$B(\underline{p})$ is an n x p matrix (elements are unknown)

The observation equation is given by

$$\underline{y}(t) = \underline{h}_x\,(\underline{x}(t)) = C\,\underline{x}(t) \tag{3.5}$$

where C is an r x n known matrix.

The argumented state vector $\underline{z}(t)$ in Equation 3.3 is expressed as

$$\underline{z}(t) = \begin{bmatrix} \dot{\underline{x}}(t) \\ \dot{\underline{p}} \end{bmatrix} = \begin{bmatrix} A(\underline{p})\ \underline{x} + B(\underline{p})\underline{u} \\ 0 \end{bmatrix} = \underline{g}_z(\underline{z}(t),\ t) \tag{3.6}$$

and the observation equation is given as

$$\underline{y}(t) = \begin{bmatrix} C \vdots 0 \end{bmatrix} \begin{bmatrix} \underline{x}(t) \\ \underline{p} \end{bmatrix} = \underline{h}_z\,(\underline{z}(t)) \tag{3.7}$$

The partial derivative of Equation 3.6 respect to $\underline{z}(t)$ is given as

$$\frac{\partial\, \underline{g}_z\,(\underline{z}(t),\ t)}{\partial\, \underline{z}(t)} = \left[\begin{array}{c|c} A & \dfrac{\partial}{\partial \underline{p}} \left[A(\underline{p})\ \underline{x} + B(\underline{p})\ \underline{u}\right] \\ \hline 0 & 0 \end{array}\right] \tag{3.8}$$

Let the sensitivity matrix $\Phi(t)$ in Equation 3.2 be partitioned as follows

$$\Phi = \left[\begin{array}{c|c} \Phi_1 & \Phi_2 \\ \hline \Phi_3 & \Phi_4 \end{array}\right] \tag{3.9}$$

then

$$\frac{d}{dt}\left[\begin{array}{c|c} \Phi 1 & \Phi 2 \\ \hline \Phi 3 & \Phi 4 \end{array}\right] = \left[\begin{array}{c|c} A & \frac{\partial}{\partial p}\left[A(p)\ \underline{X} + B(p)\ \underline{U}\right] \\ \hline 0 & 0 \end{array}\right]\left[\begin{array}{c|c} \Phi_1 & \Phi_2 \\ \hline \Phi_3 & \Phi_4 \end{array}\right] \tag{3.10}$$

It can be shown that

$$\begin{aligned} \dot{\Phi}_1 &= A\Phi_1, \quad \Phi_1(0) = I \\ \dot{\Phi}_2 &= A\Phi_2 + \frac{\partial}{\partial \underline{p}}\left[A(\underline{p})\ \underline{X} + B(\underline{p})\ \underline{U}\right], \quad \Phi_2(0) = 0 \\ \Phi_3 &= 0 \\ \Phi_4 &= I \end{aligned} \tag{3.11}$$

So we can rewrite Equation 3.9 as $$\Phi = \left[\begin{array}{c|c} \Phi_1 & \Phi_2 \\ \hline 0 & I \end{array}\right] \tag{3.12}$$

Let's compute $\frac{\partial\, \underline{h}_z(\underline{z}(t))}{\partial\, \underline{z}(t)}$ in Equation 3.1.

From Equation 3.7 $\underline{h}_z(\underline{z}(t)) = \left[\begin{array}{c|c} C & 0 \end{array}\right] \underline{z}(t)$

$$\frac{\partial\, \underline{h}_z(\underline{z}(t))}{\partial\, \underline{z}(t)} = \left[\begin{array}{c|c} C & 0 \end{array}\right] \tag{3.13}$$

If we substitute Equation 3.2, 3.12 and 3.13 into Equation 3.1 then

$$F(\underline{z}(0)) = \begin{bmatrix} [\,C \;\vdots\; 0\,]\,\Phi(0) \\ [\,C \;\vdots\; 0\,]\,\Phi(t_1) \\ \vdots \\ [\,C \;\vdots\; 0\,]\,\Phi(t_N) \end{bmatrix} = \left[\begin{array}{cc|cc} C\,\Phi_1 & (0) & C\,\Phi_2 & (0) \\ C\,\Phi_1 & (t_1) & C\,\Phi_2 & (t_1) \\ \vdots & & \vdots & \\ C\,\Phi_1 & (t_N) & C\,\Phi_2 & (t_N) \end{array}\right] \quad (3.14)$$

where $\Phi_1(t)$ and $\Phi_2(t)$ are defined previously in Equation 3.11.

B. Singularity of the Jacobian Matrix

We want to show that if the system model to be identified is not completely observable system, then $\left[F^T(\underline{z}(0))\,W\,F\,(\underline{z}(0))\right]$ in Equation 2.12 is not invertible.

It has been shown (14) that the necessary and sufficient conditions for complete observability of the given system 3.6 and 3.7 is that columns of $Ce^{A(t-to)}$ be linearly independent for all t in $[t_o, t_N]$; therefore, if the system is not observable then the columns of $C\,e^{A(t-t_o)}$ are not linearly independent. From Equation 3.11 $\Phi_1(t) = e^{A(t-t_o)}I$ therefore $C\,\Phi_1(t) = C\,e^{A(t-t_o)}$. Hence, the columns of $C\ \ _1(t)$ are not linearly independent. Therefore the columns of $F(\underline{z}(0))$ are not linearly independent if the system described by Equations 3.4 and 3.5 is not completely observable.

We want to prove that if the column vectors of $F(\underline{z}(0))$ are not linearly independent then the column vectors of $F^T(\underline{z}(0))\,W\,F\,(\underline{z}(0))$ are

also not linearly independent. Let us denote the dimension of the vector space generated by the columns of F by rank $[F]$. The following theorem is based on (15, 16).

Theorem 3.1: rank $[F]$ = rank $[F^T W F]$.

Proof: If $\underline{a}$ is a column vector such that $F\underline{a} = 0$ then $F^T W F\underline{a} = 0$. Conversely, if $F^T WF\underline{a} = 0$, then $\underline{a}^T F^T WF\underline{a} = 0$. Hence, $F\underline{a} = 0$ since W is a positive definite matrix. Hence every vector orthogonal to F is also orthogonal to $F^T WT$. Therefore rank $[F]$ = rank $[F^T WF]$.

In summary we have shown that if the system described by Equation 3.6 and 3.7 is not completely observable, then $F^T(\underline{z}(0))W\ F(\underline{z}(0))$ in Equation 2.12 is not invertible.

C. Local Stability in the Absence of Measurement Noise

An ideal iteration scheme for curve-fitting should not only be convergent, but also have the property that as a minimizing value for the parameter vector is approached, the ratio of the computed parameter error to the true error tends to unity for each parameter. That is, if $\underline{z}_o(0)$ is vector that $J(\underline{z}_o(0))$ is locally minimum, and $\triangle \underline{z}^i(0)$ is the parameter change vector computed at the ith iteration, then it is desirable that

$$\lim_{i \to \infty} \frac{\triangle \underline{z}\ (0)}{\underline{z}^i(0) - \underline{z}_o(0)} = - I \qquad (3.15)$$

where I is a unit matrix.

An equivalent statement is

$$\left.\frac{\partial \Delta \underline{z}_o(0)}{\partial \underline{z}(0)}\right|_{z(0) = z_o(0)} = -I \qquad (3.16)$$

An iteration procedure which has this property will be termed "asymptotically efficient".

The following theorem is provided by McGhee in Reference (17). Theorem 2: Suppose that the sum squared error function, $J(\underline{z}(0))$ of Equation 2.7, takes on the value zero at $\underline{z}(0) = \underline{z}_o(0)$. Assume also that $\underline{f}(\underline{z}(0))$ in Equation 2.5, and all of its first partial derivatives posses a uniformly convergent Taylor series in an ϵ - neighborhood of $\underline{z}_o(0)$. Then, providing that the Jacobian matrix, $F(\underline{z}(0))$, is of full rank everywhere in the same neighborhood, it follows that

$$\left.\frac{\partial \Delta \underline{z}^i(0)}{\partial \underline{z}(0)}\right|_{\underline{z}(0) = \underline{z}_o(0)} = -I \qquad (3.17)$$

This theorem shows that Gauss-Newton iteration possesses extraordinarily sharp convergence properties when the conditions of this theorem are satisfied.

IV. IDENTIFIABLE STRUCTURE

A. Identifiability

The definition of identifiability of parameters of a deterministic linear system is discussed by Lee (18), Aoki (19), Staley and Yue (20), Bellman and Astrom (21). In their discussions deterministic identifiability is defined as capability of recovering the structual matrices of the system uniquely from input-output data. In stochastic systems when the output is corrupted by noise, Tse and Anton (22) define the identifiability of parameters in terms of consistency of the parameter estimator.

A definition of identifiability more suitable to this study can be found in Glover and Willems (23). Much of the following discussion is based on (23).

Consider the standard linear dynamic system:

$$\frac{d}{dt}\,\underline{x}(t) = A\,\underline{x}(t) + B\,\underline{u}(t),\quad \underline{y}(t) = C\,\underline{x}(t) + D\,\underline{u}(t) \qquad (4.1)$$

where $\underline{x}\in R^n$, $\underline{u}\in R^m$, $\underline{y}\in R^p$, $A\in R^{nxn}$, $B\in R^{nxp}$, $C\in R^{rxn}$, $D\in R^{rxp}$.
In practical identification problems such equations may often be postulated on the basis of a priori knowledge on the structure and physics of the system, with the elements of the matrices A, B, C, D, either zero, known physical constants, or certain known functions of unknown parameters. Thus if the unknown parameters are denoted by $\underline{\alpha}\in\ \Omega\subset R^q$, then the system matrices may be written as $A(\underline{\alpha})$, $B(\underline{\alpha})$, $C(\underline{\alpha})$, and $D(\underline{\alpha})$, where $A: R^q\rightarrow R^{nxn}$, $B:R^q\rightarrow R^{nxp}$, $C:R^q \rightarrow R^{nxr}$, and $D:R^q\rightarrow R^{rxp}$.

A natural question which arises in the context of such identification problems is whether or not these unknown parameters, (A, B, C, D)($\underline{\alpha}$), can be identified from observations of the system.

Much work has recently been devoted to the problem of finding canonical forms for linear systems in state space form (23, 24, 25). Thus it is well known that global canonical forms exist only in the single output cases. Otherwise, a finite combinatorial number of such forms are need to represent all minimal n-dimensional systems. Canonical parametrizations are useful (and necessary) when there is a little a priori knowledge except, perhaps, the system order. An alternative approach can be used when there is sufficient a priori information from, for example, physical considerations, to write down the system matrices as functions of relatively few unknown parameters, $\underline{\alpha}$, as (A, B, C, D)($\underline{\alpha}$). The advantages of such models that the prior knowledge is conveniently summarized and the resulting state variables and parameters have a physical interpretation. Thus identifiability of these parametrizations will be discussed in this section. Canonical forms will be discussed in next section.

A parametrization of the system matrices (A, B, C, D) is then a C^1 (i.e., continuously differentiable on Ω) function (A, B, C, D) ($\underline{\alpha}$): $\Omega \subset R^q \rightarrow R^{n(n+m+p)+mp}$. That is the system matrices are parametrized by the unknown parameters $\underline{\alpha}$.

Definition 4.1: Let (A, B, C, D) (α): $\Phi \subset R^q \rightarrow R^N$ ($N = n(n + m + p) + mp$), be a parametrization of the system matrices (A, B, C, D)

of a linear dynamical system such as Equation 4.1. This parametrization is said to be locally identifiable from the transfer function at the point $\underline{\alpha} \in \Omega$ if there exists $\epsilon > 0$ such that,

1) $\|\underline{\alpha} - \hat{\underline{\alpha}}\| < \epsilon$, $\|\underline{\beta} - \hat{\underline{\alpha}}\| < \epsilon$, $\underline{\alpha}, \underline{\beta} \in \Omega$

and 2) $C(\underline{\alpha})(Is - A(\underline{\alpha}))^{-1} B(\underline{\alpha}) + D(\underline{\alpha}) = C(\underline{\beta})(Is - A(\underline{\beta}))^{-1} B(\underline{\beta}) + D(\underline{\beta})$

for all $s \in \mathbb{C}(s \neq \lambda(A(\underline{\alpha})), \lambda(A(\underline{\beta}))$ where $\mathbb{C}$ = field of complex numbers and $\lambda(A)$ = set of all eigenvalues of A, together imply $\underline{\alpha} = \underline{\beta}$.

In other words, in a neighborhood of $\hat{\underline{\alpha}}$, there are no two systems with distinct parameters, which have a same transfer function.

A disadvantage of the above concept of local identifiability is that the nominal values, $\hat{\underline{\alpha}}$, must be known and the size of the neighborhood of $\hat{\underline{\alpha}}$ is in general not easily found. It is thus desirable to introduce the definition of global identifiability.

Definition 4.2: Let $(A, B, C, D)(\underline{\alpha}): \Omega \subset R^q \rightarrow R^{n(n+m+p)+mp}$ be a parametrization of the system matrices (A, B, C, D). This parametrization is said to be globally identifiable from the transfer function if, 1) $C(\underline{\alpha})(Is - A(\underline{\alpha}))^{-1} B(\underline{\alpha}) + D(\underline{\alpha}) = C(\underline{\beta})(Is - A(\underline{\beta}))^{-1} B(\underline{\beta}) + D(\underline{\beta})$ for all $s \in \mathbb{C}(s \neq \lambda(A(\underline{\alpha})), \lambda(A(\underline{\beta}))$, and 2) $(A, B, C, D)(\underline{\alpha})$ is minimal, together imply that $\underline{\alpha} = \underline{\beta}$.

The following sufficient condition for global identifiability from the transfer function is provided by Glover and Willems (23).

Theorem 4.1: If the equations $TA(\underline{\alpha}) = A(\underline{\beta})T$, $TB(\underline{\alpha}) = B(\underline{\beta})$,

$C(\underline{\alpha}) = C(\underline{\beta})T$, $D(\underline{\alpha}) = D(\underline{\beta})$ have a unique solution for all $\underline{\alpha}, \underline{\beta} \in \Omega$ and $T \in GL(n) = \langle T \mid \det T \neq 0 \rangle$, and the system is minimal then global identifiability is implied.

The above theorem says that the transformation matrix T should be checked whether or not it is unique, that is, the identity matrix.

B. Canonical Parametrizations

We consider the problem of what parametrizations of linear dynamical systems are appropriate for identification when there is little a priori knowledge except, perhaps, the system order. Canonical parametrizations are useful in the case of pure input/output modeling which is often called the blackbox approach. If there are several parameter sets which represent the same system and their boundary of the identifiable regions are close to each other, then there might be computational difficulties in parameter identification. Thus a system structure should be chosen not to include such parameter sets. With this in mind, a canonical form will be defined.

Definition 4.3: The multiple-input/single-output identification canonical form is specified by

$$A(\underline{\alpha}) = \begin{bmatrix} 0 & 1 & 0 & - & - & - & - & 0 \\ 0 & 0 & 1 & & & & & 0 \\ & & & & & & & \\ 0 & 0 & 0 & & & & & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & & & & & \alpha_n \end{bmatrix}, \tag{4.2}$$

$$B(\underline{\alpha}) = \begin{bmatrix} \alpha_{n+1} & \alpha_{n+n+1} & - - - - & \alpha_{pn+1} \\ & & & \\ \alpha_{n+n} & & & \alpha_{n+pn} \end{bmatrix} \qquad (4.3)$$

$$C(\underline{\alpha}) = \begin{bmatrix} 1 & 0 & 0 & - - - - & \end{bmatrix} \qquad (4.4)$$

where α_i's are the q(= n + pn) parameters to be identified.

A canonical form for a class of multiple-input/multiple-output systems will be presented. Unlike the canonical form for the single-output system, the corresponding canonical forms for multivariable systems are not unique. Several procedures to obtain multiple input/multiple output canonical forms have been shown in (24, 25 and 26). The development shown here is due to Passeri and Herget (26).

Consider the following linear, time-invariant, continuous system where the first n rows of the observability matrix are linearly independent.

$$\begin{aligned} \frac{d}{dt}\underline{x}(t) &= A\,\underline{x}(t) + B\,\underline{u}(t) \\ \underline{y}(t) &= C\,\underline{x}(t) \end{aligned} \qquad (4.5)$$

where $\underline{x}(t)$ is an n x 1 state vector

$\underline{y}(t)$ is an p x 1 output vector

$\underline{u}(t)$ is an r x 1 input vector

A is an n x n matrix

B is an n x r input matrix

C is an p x n matrix

Definition 4.4: The multiple input/multiple output identification canonical form is specified by,

$$A_c(\underline{\alpha}) = \begin{bmatrix} 0 \ldots 0 & 1 & 0 \ldots 0 \\ 0 \ldots 0 & 0 & 1 \ldots 0 \\ & & \\ 0 \cdots 0 & 0 & \cdots 1 \\ \alpha_1 \cdots \alpha_p & \alpha_{p+1} & \cdots \alpha_n \\ \alpha_{(p-1)n+1} \ldots & \ldots & \alpha_{np} \end{bmatrix} \tag{4.6}$$

$$B_c = \begin{bmatrix} b_{11} \ldots b_{1r} \\ \vdots \\ b_{n1} \ldots b_{nr} \end{bmatrix} \tag{4.7}$$

$$C_c = \left[\begin{array}{c|c} I_p & 0 \end{array}\right] \tag{4.8}$$

where $m = n - p$.

The canonical form is devised so that the first n rows of the observability matrix are not only linearly independent but identically equal to the n x n identity matrix,

$$Q^T = \begin{bmatrix} C_c \\ C_cA_c \\ C_cA_c \\ \cdot \\ \cdot \\ \cdot \\ C_cA_c^{n-p} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & . & . & . & 0 \\ 0 & 1 & 0 & . & . & . & 0 \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ 0 & 0 & 0 & . & . & . & 1 \\ x & x & x & x & x & x & x \\ x & x & x & x & x & x & x \end{bmatrix} \qquad (4.9)$$

This can be done if the first n row of the observability matrix of the system in Equation 4.5 are linearly independent.

The procedure to obtain the canonical form will be shown below. Suppose there is an invertible transformation matrix T such that $\underline{z}(t) = T\underline{x}(t)$, where $\underline{z}(t)$ is a new state vector in a canonical form representation, then we have

$$\dot{\underline{z}}(t) = TAT^{-1}\underline{z}(t) + TB\underline{u}(t) \qquad (4.10)$$

$$\underline{y}(t) = T^{-1}C\underline{z}(t)$$

Consider the observation matrix Q of the canonical form

$$Q^T = \begin{bmatrix} C_c \\ C_cA_c \\ \cdot \\ \cdot \\ \cdot \\ C_cA_c^{n-p} \end{bmatrix} = \begin{bmatrix} CT^{-1} \\ CT^{-1}TAT^{-1} \\ \cdot \\ \cdot \\ \cdot \\ CT^{-1}(TAT^{-1})^{n-p} \end{bmatrix} = \begin{bmatrix} C \\ CA \\ \cdot \\ \cdot \\ \cdot \\ CA^{n-p} \end{bmatrix} T^{-1} \qquad (4.11)$$

Comparing Equation 4.9 and Equation 4.10, we have

$$\begin{bmatrix} & \\ & T_{n \times n} \\ & \\ x\ x\ x\ x\ x\ x & \\ x\ x\ x\ x\ x\ x & \end{bmatrix} = \begin{bmatrix} C \\ CA \\ \cdot \\ \cdot \\ \cdot \\ CA^{n-p} \end{bmatrix} \qquad (4.12)$$

So we can have the required transformation matrix T.

As shown above, the procedure to obtain the canonical form is simpler than other procedures (24, 25). Note that if the system is not an observable system, then there is no invertible transformation matrix T.

Theorem 4.2: If a multiple-input/single-output system is represented by the canonical form in Definition 4.1, then the equations, $TA(\underline{\alpha}) = A(\underline{\beta})T$, $TB(\underline{\alpha}) = B(\underline{\beta})$, $C(\underline{\alpha}) = C(\underline{\beta})T$, $D(\underline{\alpha}) = D(\underline{\beta})$, have a unique solution for all $\underline{\alpha}, \underline{\beta} \in \Omega$, and $T \in GL(n)$.

Proof: From the hypothesis, $C(\underline{\alpha}) = C(\underline{\beta})T$, we have

$$\begin{bmatrix} 1\ 0\ \ldots \end{bmatrix} = \begin{bmatrix} 1\ 0\ \ldots \end{bmatrix} \begin{bmatrix} t_{11}\ t_{12}\ \ldots\ t_{in} \\ \\ t_{n1}\ t_{n2}\ \ldots\ t_{nn} \end{bmatrix}$$

Thus $t_{11} = 1$, $t_{12} = t_{13} \ldots t_{in} = 0$

Also from the hypothesis $TA(\underline{\alpha}) = A(\underline{\beta})T$

That is

$$\begin{bmatrix} 1 & 0 & \cdots & 0 \\ t_{21} & t_{22} & \cdots & t_{2n} \\ \vdots & \vdots & & \vdots \\ t_{n1} & t_{n2} & & t_{nn} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & & & \\ -\alpha_1 & -\alpha_2 & -\alpha_3 & & -\alpha_n \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ -\beta_1 & -\beta_2 & -\beta_3 & & -\beta_n \end{bmatrix} \begin{bmatrix} 1 & 0 & \cdots & 0 \\ t_{21} & t_{22} & \cdots & t_{2n} \\ & & & \\ t_{n1} & t_{n2} & \cdots & t_{nn} \end{bmatrix}$$

From the above relations, we have

$$\begin{bmatrix} 0 & 1 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} t_{21} & t_{22} & \cdots & t_{2n} \end{bmatrix}$$

$$\therefore t_{21} = 0,\ t_{22} = 1,\ t_{23} = 0 \ \ldots \ t_{2n} = 0$$

Likewise

$$\begin{bmatrix} 0 & 0 & 1 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} t_{31} & t_{32} & t_{33} & \cdots & t_{3n} \end{bmatrix}$$

$$t_{31} = 0 \quad t_{22} = 0 \quad t_{33} = 1 \quad t_{34} = 0 \ \ldots \ t_{3n} = 0$$

Similarly

$$t_{41} = 0 \quad t_{42} = 0 \quad t_{43} = 0 \quad t_{44} = 1 \ \ldots \ t_{4n} = 0$$

$$\vdots$$

$$t_{n1} = 0 \quad \ldots \quad t_{nn} = 1$$

Thus $T = \begin{bmatrix} 1 & 0 & 0 & . & . & 0 \\ 0 & 1 & 0 & . & . & 0 \\ & & & & & \\ 0 & 0 & 0 & . & . & 1 \end{bmatrix}$ and $\underline{\alpha} = \underline{\beta}$

The theorem implies that the cononical form in Definition 4.3 is globally identifiable from the transfer function.

Theorem 4.2: If a multiple-input/multiple-output system is represented by the canonical form in Definition 4.4, then the equations,

$$A_c(\underline{\alpha})T = TA_c(\underline{\beta}) \tag{4.13}$$

$$B_c(\underline{\alpha}) = TB_c(\underline{\beta}) \tag{4.14}$$

$$C_c(\underline{\alpha})T = C_c(\underline{\beta}) \tag{4.15}$$

have a unique solution for all $\underline{\alpha}, \underline{\beta} \in \Omega$ and $T \in GL(n)$.

Proof: From the Equation 4.15, we have

$$\left[\begin{array}{c:c} I_{pxp} & 0_{pxm} \end{array}\right] \begin{bmatrix} t_{11}t_{12} & . & . & . & t_{1n} \\ & & & & \\ t_{p1}t_{p2} & . & . & . & t_{pn} \\ t_{n1}t_{n2} & . & . & . & t_{nn} \end{bmatrix} = \left[\begin{array}{c:c} I_{pxp} & 0_{pxm} \end{array}\right].$$

Hence $$\begin{bmatrix} t_{11}t_{12} & . & . & . & t_{1n} \\ & & & & \\ t_{p1}t_{p2} & . & . & . & t_{pn} \end{bmatrix} = \left[\begin{array}{c:c} I_{pxp} & 0_{pxm} \end{array}\right] \tag{4.16}$$

If we substitute Equation 4.16 into Equation 4.13, then we have

$$
\left[\begin{array}{c|c}
0_{mxp} & I_{mxn} \\
\hline
\multicolumn{2}{c}{x\ x\ x\ \cdot\ \cdot\ x} \\
\multicolumn{2}{c}{x\ \cdot\ \vdots\ \cdot\ \cdot\ x}
\end{array}\right]
\left[\begin{array}{c|c}
I_{pxp} & 0_{mxp} \\
\hline
\multicolumn{2}{c}{t_{p+1,1}\ \cdot\ t_{p+1,n}} \\
\multicolumn{2}{c}{t_{n,1}\ \cdot\ t_{n,n}}
\end{array}\right]
=
\left[\begin{array}{c|c}
I_{pxp} & 0_{pxm} \\
\hline
\multicolumn{2}{c}{t_{p+1,1}\ \cdot\ t_{p+1,n}} \\
\multicolumn{2}{c}{t_{n,1}\ \cdot\ t_{n,n}}
\end{array}\right]
\left[\begin{array}{c|c}
0_{mxp} & I_{mxm} \\
\hline
\multicolumn{2}{c}{x\ x\ x\ \cdot\ \cdot\ \cdot\ x} \\
\multicolumn{2}{c}{\cdot \qquad\qquad \cdot} \\
\multicolumn{2}{c}{x\ x\ x\ \cdot\ \cdot\ \cdot\ x}
\end{array}\right]
\tag{4.17}
$$

It can be shown that the first M rows of each side of Equation 4.17 is given as

$$
\begin{array}{r}
\\ \\ m\ \text{rows} \\ \\ \\ \\ p\ \text{rows} \\ \\
\end{array}
\left[\begin{array}{cc}
t_{p+1,1} & t_{p+1,n} \\
\cdot & \\
\cdot & \\
t_{n,1} & t_{n,n} \\
\hline
x\ \cdot & x \\
\cdot\ \cdot & \cdot \\
x\ \cdot & x
\end{array}\right]
=
\left[\begin{array}{c|c}
0_{mxp} & I_{mxm} \\
\hline
x & x \\
\cdot & \cdot \\
x & x
\end{array}\right]
\begin{array}{l}
m\ \text{rows} \\ \\ p\ \text{rows} \\ \\
\end{array}
\tag{4.18}
$$

Therefore we can conclude from Equations 4.16 and 4.17 that T = I and $\underline{\alpha} = \underline{\beta}$ where I is the identity matrix.

Theorem 4.3 implies that the canonical form in Definition 4.4 is globally identifiable from the transfer function.

C. Degree of Observability

We will first examine an example of a system of ill-conditioned linear equations (27, 28, 29,30), $\underline{y} = Q^T\underline{x}$

where

$$Q^T = \begin{bmatrix} 200 & 300 \\ 100 & 149 \end{bmatrix}$$

$$\underline{y} = \begin{bmatrix} 500 \\ 249 \end{bmatrix}$$

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The solution is $x_1 = 1$, $x_2 = 1$.

Observe that if the coefficients and the measurements, $\underline{y}$, are slightly changed to

$$Q^T = \begin{bmatrix} 200 & 300 \\ 100 & 148 \end{bmatrix}, \qquad \underline{y} = \begin{bmatrix} 500 \\ 250 \end{bmatrix}$$

then the solution is $x_1 = 2.5$, $x_2 = 0.0$.

The columns of the matrix Q are nearly coincident and the matrix Q is non-singular, so the matrix Q is legally invertible. But the solution is extremely sensitive to small changes or small errors in the

coefficients and $\underline{y}$. Thus the ill-conditioned equations should be avoided whenever possible by reformulation of the problem. Ill-conditioned sets of equations are all too often the source of trouble which leads to nonconvergence of iterative solutions.

We are going to examine whether such an ill-conditioned situation can occur in estimating of initial conditions of state variables in dynamic systems. The observation matrix of a system given in Equation 4.5 is defined as

$$Q = \left[C^T \mid A^T C^T \mid (A^2)^T C^T \mid \cdots \mid (A^{n-1})^T C^T \right] \tag{4.19}$$

Brown (31, 32) proposed that the degree of independency of the columns of the matrix Q is also the degree of observability of the system. For example, if n columns of the matrix Q are orthogonal then the system will be highly observable. If there is a vector which is nearly orthogonal to all the columns of the matrix Q, then the degree of independency of the columns would be low; likewise the degree of observability for the system would be low.

Since the most orthogonal vector contains considerable information, it will be discussed quantitatively. Since we are interested in the angle between two vector rather than the length of the vectors, we normalize the columns of the matrix Q and designate it as $\underline{Q}_N$.

$$\underline{Q}_N \triangleq \left[\underline{W}_1, \cdots \underline{W}_{mn} \right] \tag{4.20}$$

where $\underline{W}i$ is an n x 1 column vector.

The development shown here is due to Brown (31, 32). We define a loss function L of the form

$$L(\underline{u}) = (\underline{W}_1^T\underline{u})^2 + (\underline{W}_2^T\underline{u})^2 + - - - - = \underline{u}^T\underline{Q}_N\underline{Q}_N^T\underline{u} \qquad (4.21)$$

where $\underline{u}$ is chosen so as to minimize L. The constraint on $\underline{u}$ may be written as

$$\underline{u}^T\underline{u} = 1 \qquad (4.22)$$

where we have taken the norm of $\underline{u}$ to be unity to coincide with the norms of $\underline{W}_1$, $\underline{W}_2$, - - - - $\underline{W}_{mn}$. It can be shown (31) that minimization of L results in the Equation 4.23.

$$(\underline{Q}_N\underline{Q}_N^T - \lambda I)\,\underline{u} = 0 \qquad (4.23)$$

where λ is a Lagrange multiplier and the matrix I is the identity matrix. By rearranging Equation 4.23 and premultiply both sides by $\underline{u}^T$, we have

$$\underline{u}^T(\underline{Q}_N\underline{Q}_N^T)\underline{u} = \underline{u}^T\lambda\,\underline{u} = \lambda = L \qquad (4.24)$$

This equation implies that the minimum of L corresponds to the smallest eigenvalue of the $\underline{Q}_N\underline{Q}_N^T$ matrix. Since the smallest eigenvalue is the loss function L, its value gives a measure of the system observability. Fetzer (33) has shown that the best case of observability will be when all the eigenvalues of $\underline{Q}_N\underline{Q}_N^T$ are equal.

Now we are going to investigate how the degree of observability varies with the system structure. Consider an un-excited system given in Equation 4.5 with $\underline{u}(t) = 0$. We rewrite as

$$\dot{\underline{x}}(t) = A\,\underline{x}(t) \qquad \underline{x}(0) = \underline{x}_o \tag{4.25}$$

$$\underline{y}(t) = C\,\underline{x}(t)$$

The output response due to the state variable initial conditions $\underline{x}_o$ is given as

$$\underline{y}(t) = C\,e^{At}\,\underline{x}(0) \tag{4.26}$$

By using the Caley-Hamilton theorem (30),

$$e^{At} = \sum_{k=0}^{n-1} \alpha_k(t)\,A^k \tag{4.27}$$

where n is the dimension of A, and

$$\underline{\alpha}(t) = (\alpha_o(t), \alpha_1(t), - - - , \alpha_{n-1}(t))$$

are linearly independent over any interval of positive length. The output can be written as, (4.28)

$$\underline{y}(t) = \left[\alpha_o(t)CA^o + \alpha_1(t)CA + \alpha_2(t)CA^2 + - - - \alpha_{n-1}(t)CA^{n-1}\right]\underline{x}(0)$$

The output is sampled and rewritten in the matrix form shown by Equation 4.29.

(4.29)

$$\begin{bmatrix} \underline{y}(t_o) \\ \underline{y}(t_1) \\ \cdot \\ \cdot \\ \underline{y}(t_{n-1}) \end{bmatrix} = \begin{bmatrix} \alpha_o(t_o) & \alpha_1(t_o) & & \alpha_{n-1}(t_o) \\ \alpha_o(t_1) & \alpha_1(t_1) & & \alpha_{n-1}(t_1) \\ & & & \\ & & & \\ \alpha_o(t_{n-1}) & \alpha_1(t_{n-1}) & - - - & \alpha_{n-1}(t_{n-1}) \end{bmatrix} \begin{bmatrix} CI \\ \hline CA \\ \hline \\ \\ \hline CA^{n-1} \end{bmatrix} \underline{x}(0)$$

Assume there is a similarity transformation matrix T such that

$$A_z = TAT^{-1}$$
$$C_z = CT^{-1} \tag{4.30}$$

Then, from Equation 4.25, we have

$$\dot{\underline{z}} = A_z \underline{z}, \qquad \underline{z}(0) = T \underline{x}(0) \tag{4.31}$$
$$\underline{y} = C_z \cdot \underline{z}$$

where $\underline{z}$ is a new state vector. Substituting Equations 4.30 and 4.31 into Equation 4.28, we have (4.32)

$$\begin{bmatrix} \underline{y}(t_o) \\ \underline{y}(t_1) \\ \cdot \\ \cdot \\ \underline{y}(t_{n-1}) \end{bmatrix} = \begin{bmatrix} \alpha_o(t_o) & \alpha_1(t_o) & - & - & - & -\alpha_{n-1}(t_o) \\ \alpha_o(t_1) & \alpha_1(t_1) & - & - & - & -\alpha_{n-1}(t_1) \\ & & & & & \\ & & & & & \\ \alpha_o(t_{n-1}) & & - & - & - & -\alpha_{n-1}(t_{n-1}) \end{bmatrix} \begin{bmatrix} C_z I \\ \hline C_z A_z \\ \hline \\ \\ \hline C_z A_z^{n-1} \end{bmatrix} \underline{z}(0)$$

Comparing Equations 4.29 and 4.33, we can see that the matrix

$$\begin{bmatrix} \alpha_o(t_o) & \cdot & \cdot & \alpha_{n-1}(t_o) \\ \cdot & & & \\ \cdot & & & \\ \alpha_o(t_{n-1}) & \cdot & \cdot & \alpha_{n-1}(t_{n-1}) \end{bmatrix} \tag{4.33}$$

is invariant under the similarity transformation. If the degree of observability matrix

$$Q^T = \begin{bmatrix} CI \\ CA \\ \cdot \\ CA^{n-1} \end{bmatrix} \tag{4.34}$$

in Equation 4.29 is very low, then for any sampling time, small errors in the system structure matrices A and C may cause some computational difficulty in estimating the initial condition $\underline{x}_0$. When the degree of observability is low, it can be changed by the similarity transformation shown in Equation 4.32 in order to avoid such a difficulty.

We are going to examine the degree of observability of the canonical forms given in Definition 4.3 and 4.4.

Theorem 4.4: If a single output system is represented by a canonical form in Definition 4.3, then it possesses the best case of observability.

Proof: Substitute Equations 4.2 and 4.4 into Equation 4.19, we can show that the observability matrix is given by

$$Q^T = \begin{bmatrix} C \\ CA \\ CA^2 \\ \cdot \\ \cdot \\ \cdot \\ CA^{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & - & - & - & 0 \\ 0 & 1 & 0 & - & - & - & 0 \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ 0 & 0 & 0 & - & - & - & 1 \end{bmatrix} \tag{4.35}$$

Thus, the n columns of the Q matrix are orthogonal to each other, and the eigenvalues of $\underline{Q}_N \underline{Q}_N^T$ are equal.

Theorem 4.5: If a multiple-input/multiple-output system is represented by the canonical form in Definition 4.4, then it possesses the best case of observability.

Proof: Substituting Equation 4.9 into 4.29, we have

$$\begin{bmatrix} \alpha_0(t_0)\alpha_1(t_0) & - - - - \alpha_{n-1}(t_0) \\ & \\ & \\ & \\ & \\ \alpha_0(t_{n-1})\alpha_1(t_{n-1}) & - - - \alpha_{n-1}(t_{n-1}) \end{bmatrix}^{-1} \begin{bmatrix} \underline{y}(t_0) \\ \\ \underline{y}(t_1) \\ \\ \cdot \\ \\ \cdot \\ \\ \underline{y}(t_{n-1}) \end{bmatrix} = \tag{4.36}$$

$$\begin{bmatrix} 1 & 0 & 0 & - & - & - & 0 \\ 0 & 1 & 0 & & & & \\ 0 & 0 & 1 & & & & \\ & & & & & & \\ 0 & 0 & 0 & & & & 1 \\ & & & & & & \\ x & x & x & & & & x \\ x & x & x & & & & x \end{bmatrix} \begin{matrix} \underline{x}_0 \\ \\ \\ \\ \text{n rows} \\ \\ \\ \\ \end{matrix}$$

We may consider only the first n rows among p n rows in the left-hand side of Equation 4.36 because of redundant observations.

We can define

$$Q = \begin{bmatrix} 1 & 0 & 0 & - & - & - & 0 \\ 0 & 1 & 0 & - & - & - & 0 \\ & & & & & & \\ 0 & 0 & 0 & & & & 1 \end{bmatrix}$$

Thus, there are n independent columns of the matrix Q which are orthogonal to each other, and all the eigenvalues of $\underline{Q}_N \underline{Q}_N^T$ are equal. Other multiple canonical forms shown in (24, 25) might have the best case of observability, but it is hard to prove it.

In summary, the identifiability of linear systems is discussed and the properties of parameterizations are investigated in this chapter. It has been proved in this chapter that the canonical forms given in Definitions 4.3 and 4.4 are globally identifiable from the transfer function and possess the best case of observability.

V. BOILER STRUCTURE AND LOW-ORDER BOILER MODELS

It is apparent that the physical model of a boiler turbine system is complex and is a distributed parameter system. A rigorous mathematical description would require partial differential equations. The concern about the mathematical model of the transient thermal process is a fairly recent one, dating back less than 30 years. A review of the literature on the physical modeling of a boiler is given in the recent papers (1, 2).

Boiler-turbine systems will be reviewed from the system control point of view, and proposed low-order boiler models will be investigated in this chapter. Much of the following presentation is based on (1).

A. Boiler Types and Controls

The definition of a steam generator, according to the ASME Codes, is a combination of apparatus for producing, furnishing, or recovering heat, together with apparatus for transferring to a working fluid the heat thus made available. However the term boiler has been used for such a long period of time the two terms are used interchangeably.

The fluid-flow path of a drum-type modern plant boiler-turbine is shown in Figure 5.1. Not all systems are exactly like that of Figure 5.1, which is a pulverized coal-fired, twin furnace, drum-type, controlled circulation, single reheat boiler type. The portions of the feedwater heaters to the boiler feed pump and the

economizer are not shown. Boilers may be classified in several ways, one of which is the method of water circulation: drum-type natural circulation, drum-type forced circulation, and once-through boilers.

Thermal energy storage varies greatly between drum type and once-through boilers. In once-through boilers there are more strict requirements for balanced conditions between feedwater flow and firing rate, and there is closer coupling between the feed pump and the turbine valve than in the case of drum-type units. Reliable functioning of the control devices is essential to keep the steam pressure and temperature within narrow limits because they have limited storage energy to hold specified constant pressure at the throttle valves without control effort.

There are three different types of boiler control in general use in the United States. These are usually called turbine follow, boiler follow and integrated control. The turbine following mode of control is shown in Figure 5.2. Turbine control valves are made to regulate boiler pressure and the generated power is controlled by changing the boiler inputs (fuel, air, feed-water). The fast action of the turbine control valves accomplishes almost perfect pressure control so that the boiler pressure can be maintained essentially constant. A demand for change in power generation acts on the boiler inputs and the turbine valves respond as the change in energy level is developed in the boiler. The power response is thereby delayed by the lags in the fuel system and boiler storage. Response is very slow but stable, with no overshoot in pressure.

The boiler following control mode is shown in Figure 5.3. In this mode, the generated power is changed by turbine control valves and the boiler inputs are controlled with appropriate action upon sensing the changes in steam flow and pressure. From a system control point of view, this rapid response characteristic is desirable, improving the quality of frequency control. Evidently, the rapid response can be at the expanse of overshoot in boiler controlled variables such as the boiler throttle pressure and the main steam temperature.

A compromise between the desire for fast generation response, the desire for boiler safety, and the limitation of deviations of boiler controlled variables (especially boiler pressure and temperature) led to the adoption of a control mode commonly known as coordinated or integrated control. The integrated control mode is shown in Figure 5.4. Both pressure and generated output are fed back for control of both the boiler and the turbine. The turbine valve is primarily the pressure regulator, and boiler inputs are actuated basically from load demand.

The actual steam flow to the turbine is primarily a function of the effective valve opening area and the throttle pressure. The purpose of the turbine control valve systems is to control the effective valve opening area at the high pressure turbine admission in response to the control valve flow signal. Usually a set of four control valves are employed in steam turbines for efficient valve throttling. Because of the appreciably non-linear steam flow

characteristic of the steam valves, compensation devices must be introduced to obtain linear steam flow response with respect to the control valve flow signal. Function generators can be used in the forward loop or feedback loop and can be implemented mechanically or electrically. A typical electro-hydraulic control system is shown in Figure 5.5 (34).

B. Low-order Boiler Models

A number of low-order boiler models have been proposed for the study of power plant dynamics. Many references are listed and reviewed in some recent papers. Two of the low-order models will be discussed in this section, and their structure is used in the simulation study in the next chapter.

A linear low-order model was presented in a 1971 paper by Laubli and Fenton (35). This model is for a once-through steam boiler and has a boiler following control mode. The authors show the relationship between inputs (virtual steam production W_V and valve opening area A_V) and outputs (throttle pressure D_T and steam flow rate W_T). This is illustrated in Figure 5.6(a). From the responses illustrated, the transfer function can be computed as shown Figure 5.6 (b). The complete model of the boiler, including combustion controller, is shown in Figure 5.7, and various time constants are tabulated in Table 5.1. The lag between virtual steam production and the fuel flow rate is represented by a third order transfer function. Its representation is dependent on the type

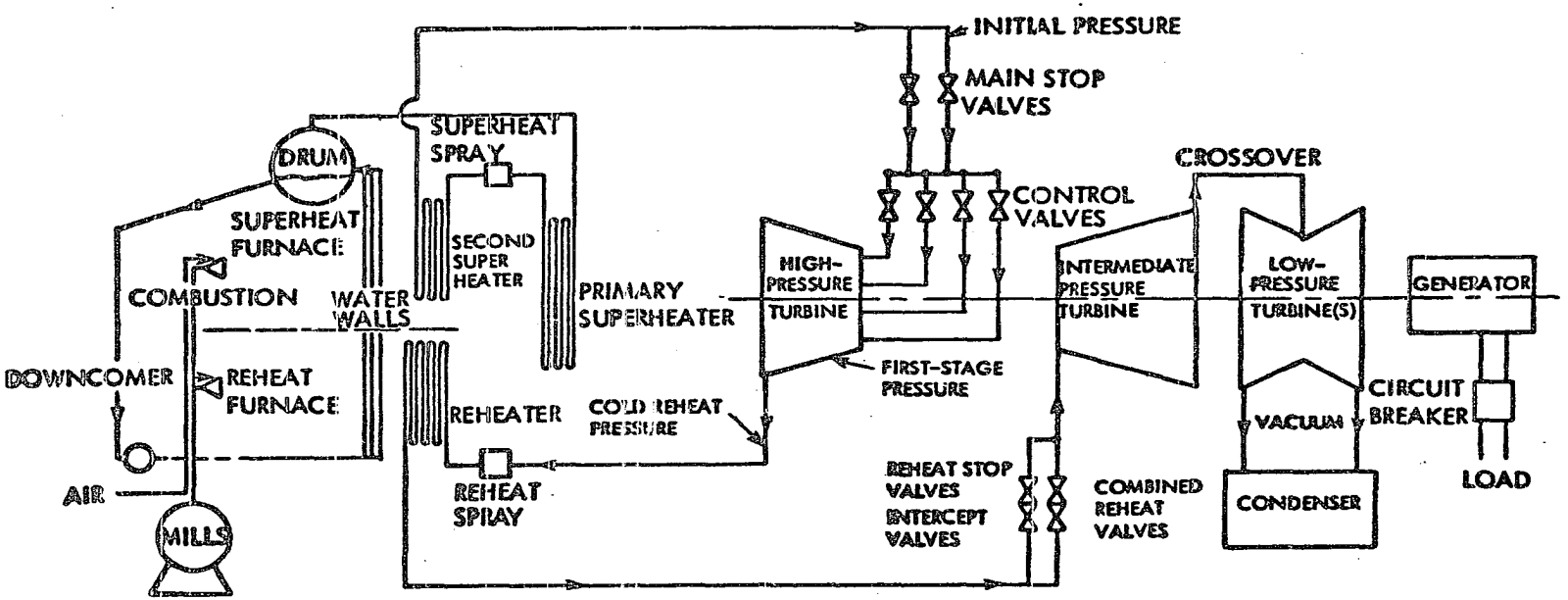


Figure 5.1. Fluid-flow path for a typical drum-type boiler and single reheat turbine

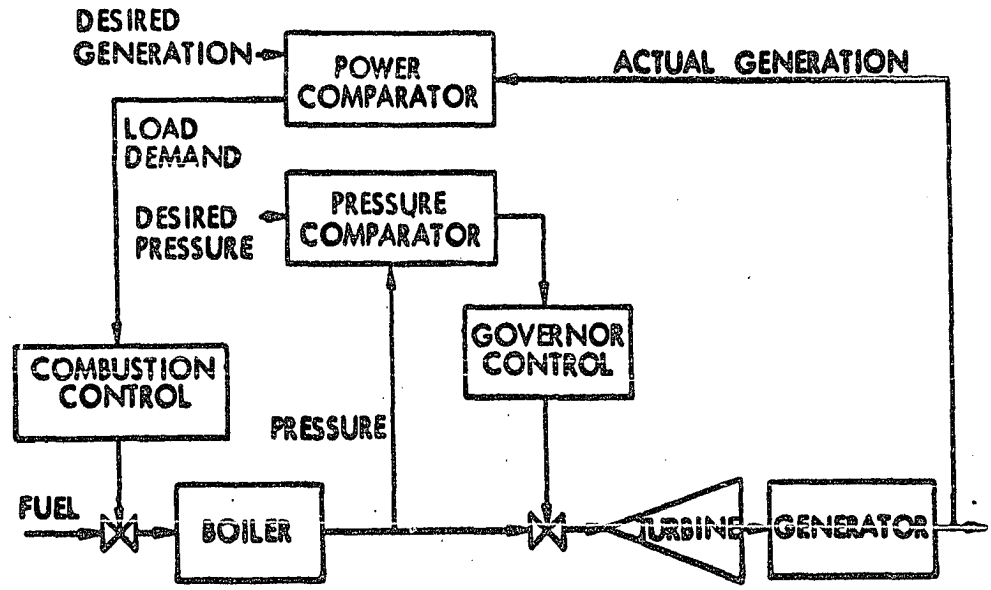


Figure 5.2. Turbine follow control system

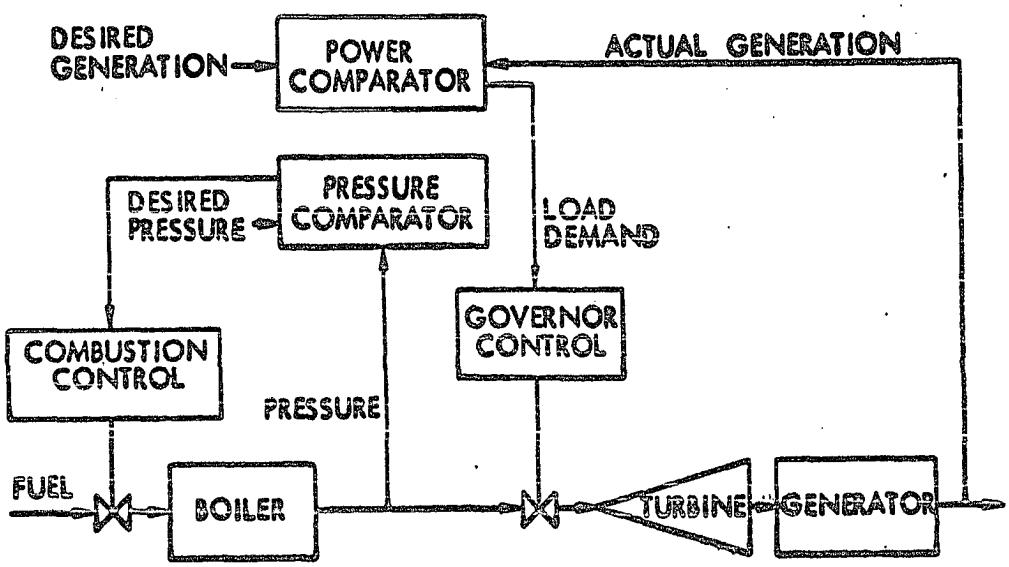


Figure 5.3. Boiler follow control system

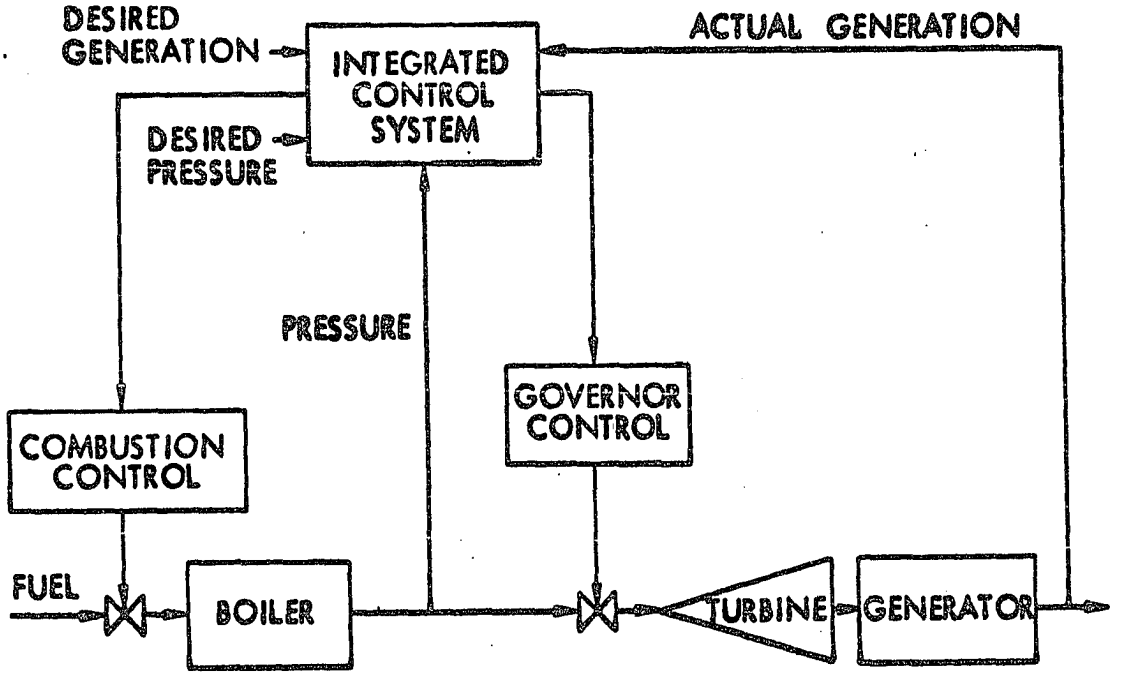


Figure 5.4. Integrated boiler-turbine control system

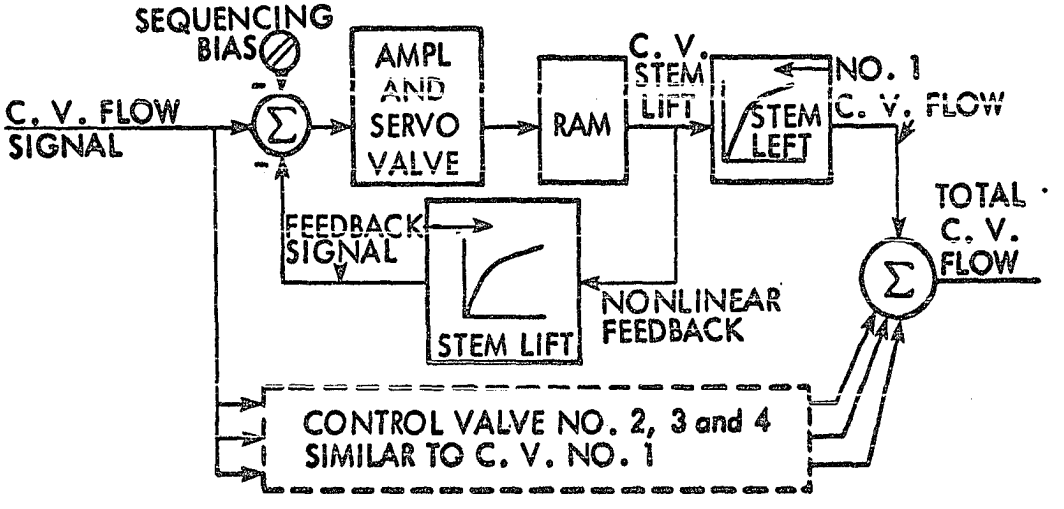


Figure 5.5. Control valve flow control unit

of fuel (gas, oil or coal) and the method of fuel preparation and feeding.

Another low-order model is that of Frensch and Klefenz (1, 36). This model is linear and for the once-through steam boiler. The pressure component of the Frensch and Klefenz model is shown in Figure 5.8. The original Frensch and Klefenz model included two main parts: a pressure model and a temperature model. The temperature components response is slow enough not to be included during the time of interest in this study. The model includes the effect of feedwater variation to the virtual steam production.

Here we define symbols which will be used to represent a boiler system as follows:

$$\Delta A_V(t) = \frac{A_V(t) - A_V(0)}{A_V(0)}$$

$$\Delta C_V(t) = \frac{C_V(t) - C_V(0)}{C_V(0)}$$

$$\Delta C_F(t) = \frac{C_F(t) - C_F(0)}{C_F(0)}$$

$$\Delta D_T(t) = \frac{D_T(t) - D_T(0)}{D_T(0)}$$

$$\Delta W_V(t) = \frac{W_V(t) - W_V(0)}{W_V(0)}$$

$$\Delta W_T(t) = \frac{W_T(t) - W_V(0)}{W_T(0)}$$

where $A_V(t)$ is the effective valve opening area,

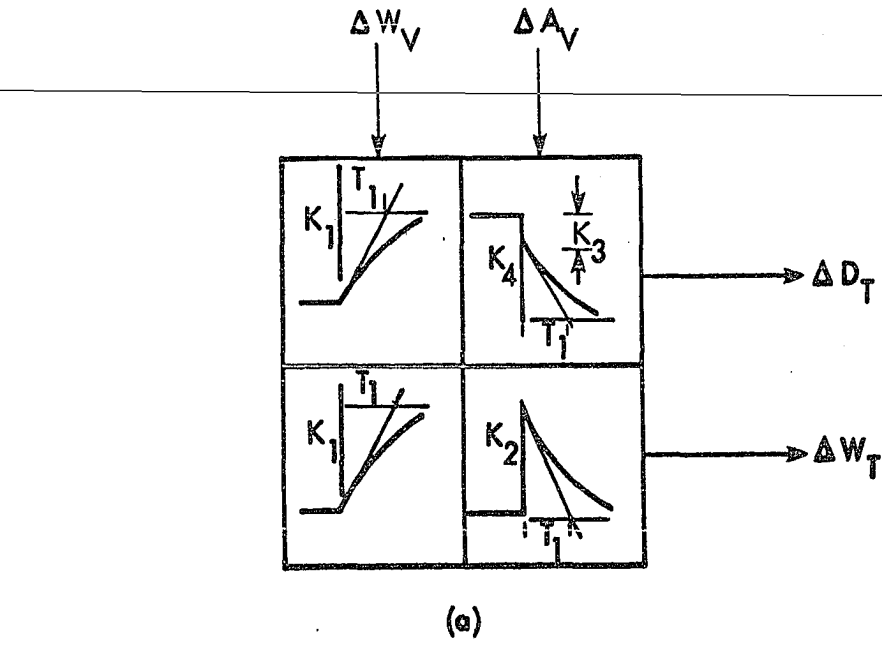
$C_V(t)$ is the control valve flow signal,

$C_F(t)$ is the fuel flow rate to the boiler,

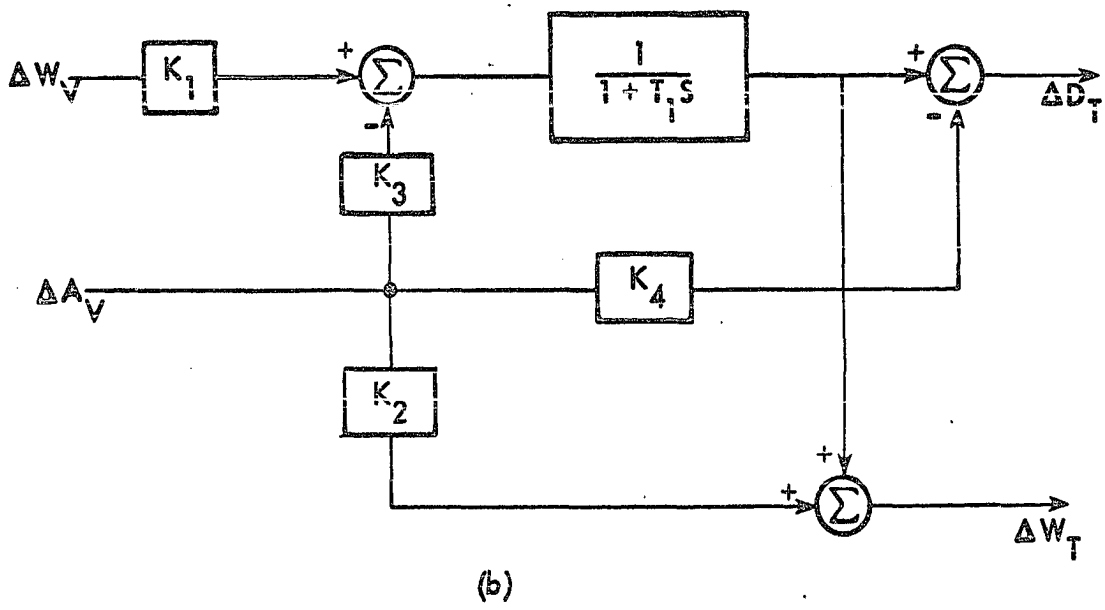
$D_T(t)$ is the throttle pressure,

$W_V(t)$ is the virtual steam production, and

$W_T(t)$ is the steam flow rate to the turbine.



(a)



(b)

Figure 5.6. The relation of W_V, A, and D_T, W_T
(a) Graphical representation
(b) Transfer function

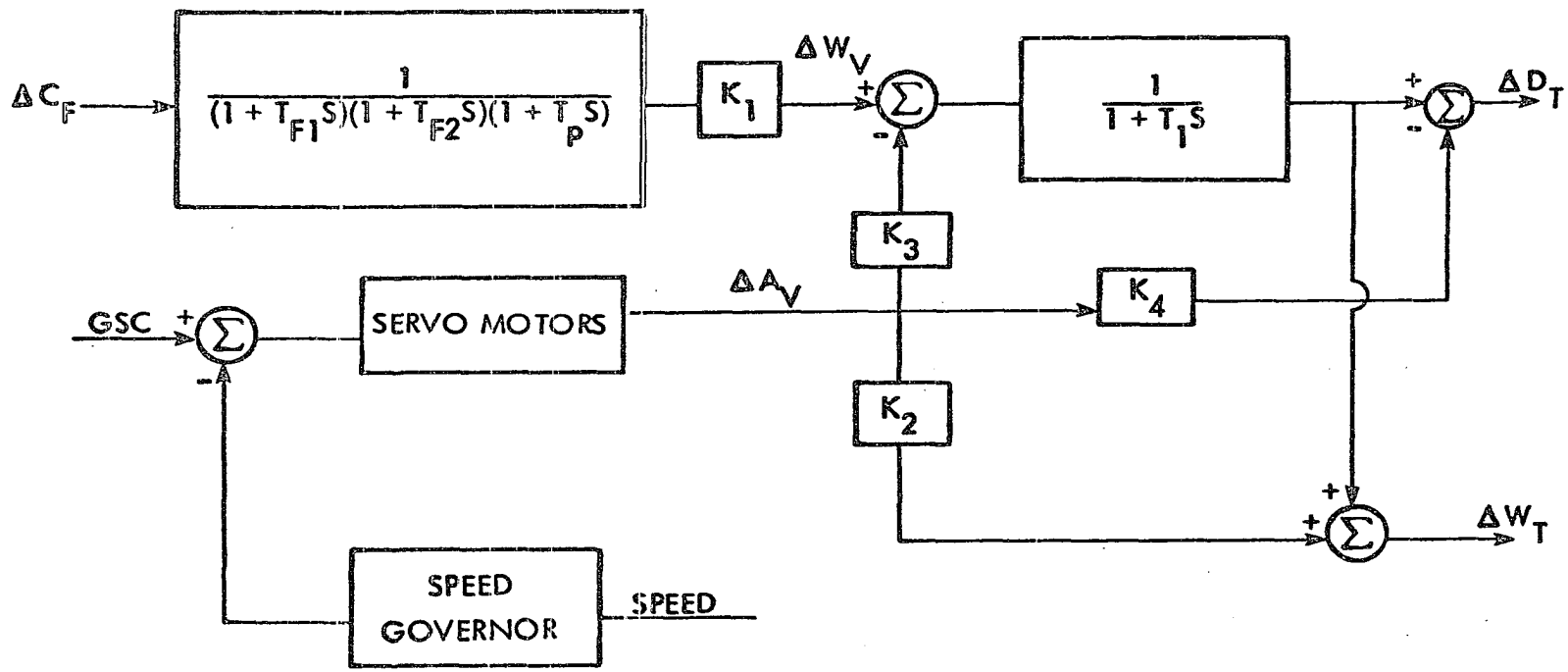


Figure 5.7. Block diagram of Laubli-Fenton Model

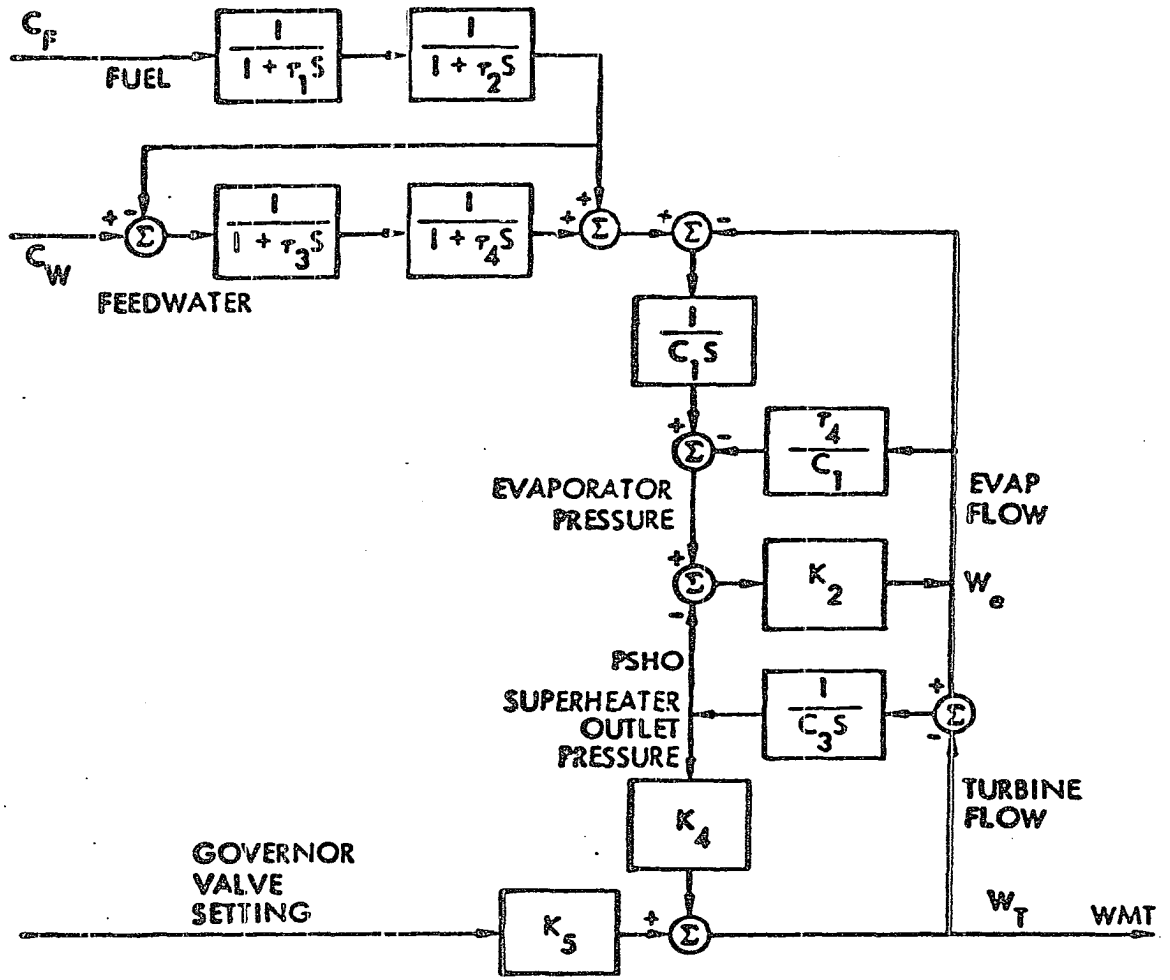


Figure 5.8. The Boiler Pressure Model of Frensch-Klefenz

Table 5.1. Typical constants for the Laubli-Fenton model of Figure 5.7

	Gas/Oil Fired	Coal Fired
T_{F1}	8.0 Sec.	40.0 Sec.
T_{F2}	0 Sec.	40.0 Sec.
T_P	6.0 Sec.	20.0 Sec.
T_1	115.0 Sec.	115.0 Sec.
K_F	4.5 Sec.	1.6 Sec.
K_D	0.15 Sec.	4.0 Sec.
T_2	45.0 Sec.	27.0 Sec.
T_D	200.0 Sec.	115.0 Sec.
K_1	1.0	1.0
K_2	0.87	0.87
K_3	0.87	0.87
K_4	0.13	0.13

Table 5.2. Typical constants for the Frensch-Klefenz model of Figure 5.8.

Gains	Time Constant
$K_1 = 1.72/44$	$T_1 = 20.0$
$K_2 = 0.2$	$T_2 = 25.5$
$K_4 = 0.027$	$T_3 = 97.0$
$K_5 = 1.0$	$T_4 = 44.0$
$C_1 = 1.72$	
$C_3 = 0.7$	

VI. SIMULATION STUDIES OF PARAMETER IDENTIFICATION OF LOW-ORDER BOILER MODELS

Usually a simulation study is made before an actual experiment is performed in order to increase the possibility of obtaining useful data and good results, especially when the experiment is rarely available. In this chapter, the application of the algorithm developed in the previous chapters has been made to the estimation of parameters in low-order boiler models under simulated operating conditions.

A. Simulated Boiler Input/Output Data

The actual boiler input/output data are generated by the simulation of a linearized electric system model (35). As shown in Figure 6.1, the electric power system model includes the boiler, turbine, electric grid, frequency control, and firing control models. The electric system model does not include the excitation system for which the transient response time constants are generally no longer than one or two seconds.

The actual boiler model, which is a higher order model than the model to be identified, is assumed to be Frensch and Klefenz (1, 36). The boiler control mode of the Frensch and Klefenz model is assumed to be a boiler follower and the response of the virtual steam production (W_V) is assumed to be that of the model to be identified, but the parameters are unknown. The boiler has a coal-firing system and is a once-through boiler type.

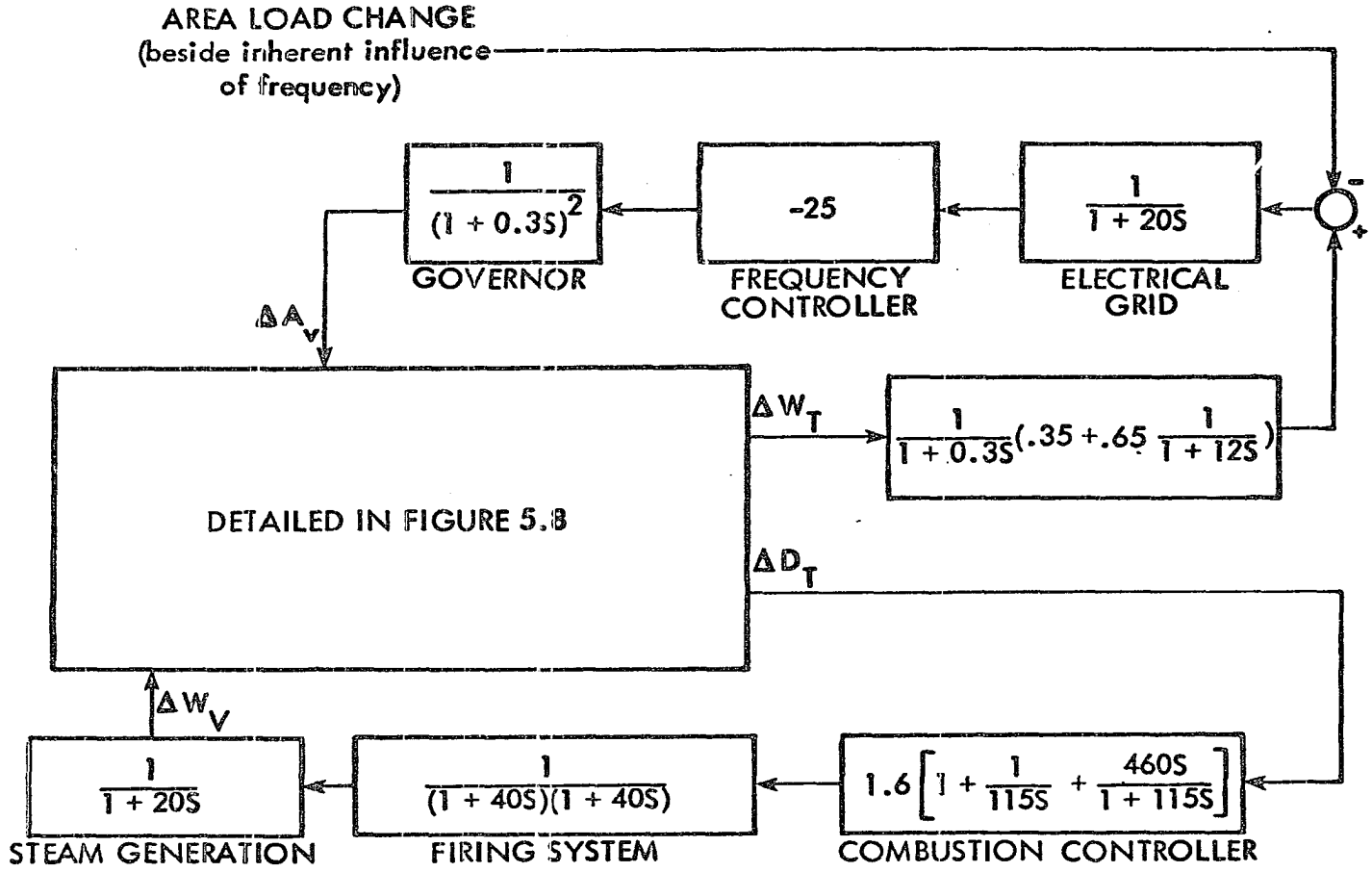


Figure 6.1. Block Diagram of an Electric Power System

A sinusoidal change in power consumption is assumed in order to disturb the electric system. Usually it is not known how loads vary with voltage and frequency. But it is assumed that there is no inherent influence of frequency on assumed area load change. The load change has a time period 100 seconds, the disturbance observation period is 400 seconds, and the maximum amplitude and the minimum amplitude are limited by +0.1 pu and -0.1 pu respectively. The sampling interval was chosen as 4 seconds.

B. The Model to be Identified

The choice of structure of the linear model for the drum boiler-turbine unit is based on the Laubli and Fenton boiler model (35). The model is 4th order system and has two inputs (valve opening area, A_V, and coal firing rate, C_F) and two outputs (throttle pressure, D_T, and steam flow rate, W_T).

Since the low-order model is the representation of the boiler dynamics which is initially at rest and then followed by a disturbance, the state variable initial conditions can be assumed to be known; however, they are not known in general. Both cases are treated in the simulated experiments.

C. Precomputation for the Identification Algorithm Structure A

The Laubli-Fenton boiler model which is to be identified can be represented by the following state variable form as

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{u}(t)$$

$$\underline{y}(t) = C\underline{x}(t) + D\underline{u}(t) \qquad (6.1)$$

where $\underline{x}(t)$ is a 4x1 state vector,

$\underline{y}(t)$ is a 2x1 output vector:

y_1 = throttle pressure, D_T

y_2 = steam flow rate, W_T

$\underline{u}(t)$ is a 2x1 input vector:

u_1 = valve opening area, A_V

u_2 = coal firing rate, C_F

The matrices A, B, C and D are given as follows:

$$A = \begin{bmatrix} -a & a & 0 & 0 \\ 0 & -b & b & 0 \\ 0 & 0 & -c & c \\ 0 & 0 & 0 & -d \end{bmatrix}$$

$$B = \begin{bmatrix} -K_3 x a & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & K_1 x d \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} -K_4, & 0 \\ K_2, & 0 \end{bmatrix}$$

where $a = \frac{1}{T_1}$, $b = \frac{1}{T_P}$, $c = \frac{1}{T_{F1}}$ and $d = \frac{1}{T_{F2}}$.

Some elements of the matrices A, B, and D are assumed unknown, that is, T_1, T_P, T_{F1}, T_{F2}, K_1, K_2, K_3 and K_4 are unknown. It is helpful for finding a canonical form to consider this model as a single output system since two rows in the C matrix are not independent. From Equation 4.12, we can find a linear transformation matrix T which transforms into A into the identifiable cannonical form.

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -a & a & 0 & 0 \\ a^2 & -a^2 - ab & ab & 0 \\ -a^3 & a^3 + a^2b + ab^2 & -a^2b - ab^2 - abc & abc \end{bmatrix} \tag{6.2}$$

If we represent the system including the parameters in the cannonical form, then new state variable for follows as

(6.3)

$$\frac{d}{dt}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -z_5 & -z_6 & -z_7 & -z_8 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} -z_9 \times z_{10} & 0 \\ z_9^2 \times z_{10} & 0 \\ -z_9^3 \times z_{10} & 0 \\ z_9^4 \times z_{10} & Z_{11} \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\underline{y}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -u_1(t) & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u_1(t) \end{bmatrix} \underline{z}(t)$$

(6.4)

where: z_1, z_2, z_3, z_4 are new state variables whose initial conditions are unknown.

$$z_5 = abcd$$

$$z_6 = ab + bc + cd + da + ac + bd$$

$$z_7 = abc + bcd + cda + dab$$

$$z_8 = a + b + c + d$$

$$z_9 = a$$

$$z_{10} = K_3$$

$$z_{11} = abcdK_1$$

$$z_{12} = K_4$$

$$z_{13} = K_2$$

The sensitivity functions $\Phi(t)$ are computed as follows:

$$\dot{\Phi}(t) = \frac{\partial \underline{g}_z(z(t).t)}{\partial \underline{z}(0)} \Phi(t), \qquad \Phi(0) = I \qquad (6.5)$$

where $\dfrac{\partial g_z(z(t).t)}{\partial \underline{z}(0)} =$

$$\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -3z_{10}u_1, & z_9 u_1, & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2z_9 z_{10}u_1, & z_9^2 u_1, & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -3z_9^2 z_{10}u_1, & -z_{10}^3 u_1, & 0 & 0 & 0 \\
-z_5 & -z_6 & -z_7 & -z_8 & -z_1 & -z_2 & -z_3 & -z_4 & 4z_9^3 z_{10}u_1, & u_2, & 0 & 0 & 0 \\
 & & & & & & 0 & & & & & &
\end{bmatrix}$$

The matrix of sensitivity functions, $\Phi(t)$, consists of 13x13(=169) differential equations; however, if we only consider non-zero elements, then we can reduce this number to 44 differential equations. The 44 sensitivity function differential equations plus 13 other dynamic system equations were solved by Hamming's modified predictor-corrector method, using the scientific subroutine package of the IBM computer.

Structure B

If we represent the identification model by another canonical form which is close to the structure of the Laubli-Fenton model, then we can have a new form as follows:

$$\frac{d}{dt}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} -z_5 & z_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -z_7 & -z_8 & -z_9 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} -z_{10} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (6.6)$$

$$\underline{y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -u_1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u1 \end{bmatrix} \underline{z} \qquad (6.7)$$

where $\underline{y}$ is an 2 x 1 output vector

$\underline{z}$ is an 12 x 1 state vector

$z_5 = 1/T_1$

$z_6 = K_1/T_1$

$z_7 = 1/(T_{F2} \times T_{F1} \times T_p)$

$$z_8 = 1/(T_{F1} x T_{F2}) + 1/(T_{F2} x T_p) + 1/(T_p x T_{F1})$$

$$z_9 = 1/T_{F1} + 1/T_{F2} + 1/T_p$$

$$z_{10} = K_2/T_1$$

$$z_{11} = K_4$$

$$z_{12} = K_2$$

$$\underline{u} = (u_1, u_2)^T$$

where $u_1 = \Delta A_V$,

$u_2 = \Delta C_F$

The sensitivity function, $\Phi(t)$, is given by

$$\dot{\Phi}(t) = \frac{\partial \underline{g}(\underline{z},t)}{\partial \underline{z}(0)} \Phi(t), \qquad \Phi(0) = I \tag{6.8}$$

where

$$\frac{\partial \underline{g}(\underline{z},t)}{\partial \underline{z}(0)} = \begin{bmatrix} -z_5 & z_6 & 0 & 0 & -z_1 & -z_2 & 0 & 0 & 0 & -u_1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & z_7 & -z_8 & -z_9 & 0 & 0 & -z_2 & -z_3 & -z_4 & 0 & 0 & 0 \\ & & & & & & & & & & & \\ & & & & & 0 & & & & & & \end{bmatrix} \tag{6.9}$$

D. Parameter Identification Computer Program

This section describes the computational procedures of the experiments for the estimation of the parameters of the model

structures A and B, given in Equations 6.3, 6.4, 6.6 and 6.7, from simulated input/output data. The computational policy is chosen as follows:

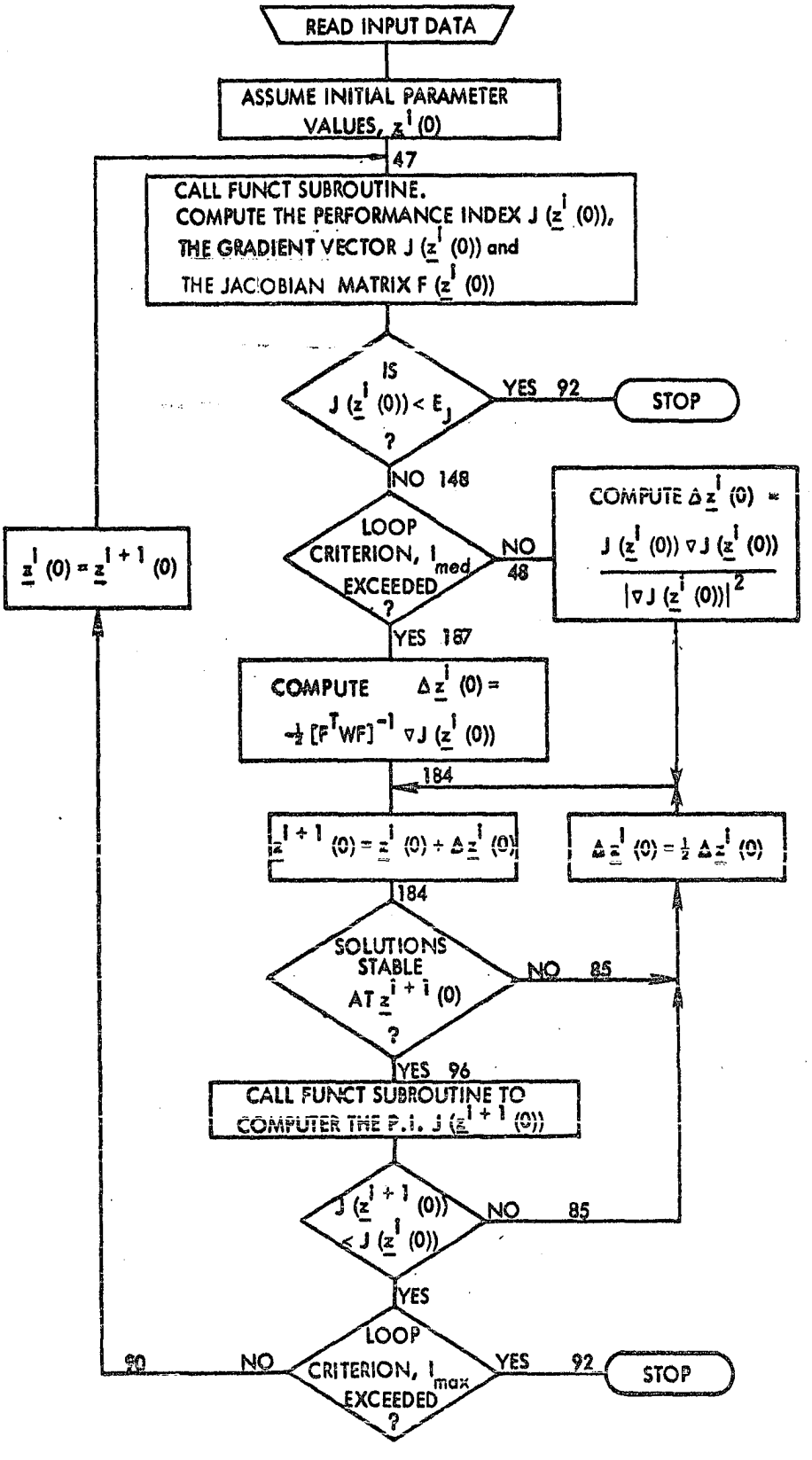
1. The Newton-Raphson iteration method is used for the initial states of iteration when there is little a priori information about the initial parameter values.
2. The Gauss-Newton iteration method is mainly used in the terminal stages of iteration. If the Gauss-Newton parameter change vector, $\Delta \underline{z}^i(0)$ in Equation 2.11 and 2.17, provides a reduction in the performance index, $J(\underline{z}^i(0))$ in Equation 2.10, at the i-th stage of iteration, then $\underline{z}^{i+1}(0) = \underline{z}^i(0) + \Delta \underline{z}^i(0)$ is used.
3. If this is not so, then $\underline{z}^{i+1}(0)$ is determined by a binary search along the change vector, $\Delta \underline{z}^i(0)$, for a local minimum value of $J(\underline{z}^{i+1}(0))$.

The implementation of this policy is shown in Figure 6.2, and the computer program is included in Appendix A.

The operation of the program can be explained as follows by referring to Figure 6.2.

1. All data are first read in and stored.
2. Initial parameters $\underline{z}^i(0)$ are chosen
3. The FUNCT subroutine computes the performance index $J(\underline{z}^i(0))$ in Equation 2.7, and the Jacobian matrix $F(\underline{z}^i(0))$ in Equation 3.1, and the gradient vector $\nabla J(\underline{z}^i(0))$ in Equation 2.13.

Figure 6.2. Flow diagram of parameter identification program


READ INPUT DATA
ASSUME INITIAL PARAMETER VALUES, $\underline{z}^1(0)$
47
CALL FUNCT SUBROUTINE.
COMPUTE THE PERFORMANCE INDEX J ($\underline{z}^1(0)$),
THE GRADIENT VECTOR J ($\underline{z}^1(0)$) and
THE JACOBIAN MATRIX F ($\underline{z}^1(0)$)
IS $J(\underline{z}^1(0)) < E_J$?
YES 92
STOP
NO 148
LOOP CRITERION, I_{med} EXCEEDED ?
NO 48
COMPUTE $\Delta \underline{z}^1(0) = \frac{J(\underline{z}^1(0))\,\nabla J(\underline{z}^1(0))}{|\nabla J(\underline{z}^1(0))|^2}$
YES 187
COMPUTE $\Delta \underline{z}^1(0) = -\frac{1}{2}[F^T W F]^{-1} \nabla J(\underline{z}^1(0))$
$\underline{z}^1(0) = \underline{z}^{1+1}(0)$
184
$\underline{z}^{1+1}(0) = \underline{z}^1(0) + \Delta \underline{z}^1(0)$
$\Delta \underline{z}^1(0) = \frac{1}{2} \Delta \underline{z}^1(0)$
184
SOLUTIONS STABLE AT $\underline{z}^{i+1}(0)$?
NO 85
YES 96
CALL FUNCT SUBROUTINE TO COMPUTER THE P.I. $J(\underline{z}^{1+1}(0))$
$J(\underline{z}^{1+1}(0)) < J(\underline{z}^1(0))$
NO 85
YES
LOOP CRITERION, I_{max} EXCEEDED ?
80
NO
YES 92
STOP

4. A check is made to determine if the performance index $J(\underline{z}^i(0))$ is less than the given value, E_J. If so, the procedure is stopped and the present estimator, $\underline{z}^i(0)$, is assumed to be identified. If not, follow next procedure.
5. The Newton-Raphson iteration method is implemented for the initial stages, and the Gauss-Newton iteration method is implemented for the terminal stages. The change vector, $\triangle \underline{z}^i(0)$, given in Equation 2.19, is computed if the number of iteration does not exceed the predetermined number I_{med}. If so, the change vector, $\underline{z}^i(0)$, is computed according to Equations 2.15 and 16.
6. The new estimate $\underline{z}^{i+1}(0)$ is computed.
7. The new estimate is used to check whether the system differential equations are stable at the new estimate.
8. If the equations are not stable, then the change vector, $\triangle \underline{z}^i(0)$, is reduced and the procedure is repeated from #6.
9. The performance index, $J(\underline{z}^{i+1}(0))$, is computed at the new estimate, $\underline{z}^{i+1}(0)$, and checked if $J(\underline{z}^{i+1}(0))$ is reduced. If not, the change vector $\triangle \underline{z}^i(0)$ is reduced by half, and the procedure is repeated from #6.
10. If so, a check is made to determine whether the number of iterations exceeds the predetermined maximum iteration number, I_{max}.

11. If the iteration number exceeds I_{max}, then the present estimate, $\underline{z}^{i+1}(0)$, is assumed to be the identified parameter vector. If not, the iteration process is re-initiated as follows: use $\underline{z}^{i+1}(0)$ to replace the initial guess $\underline{z}^{i}(0)$ and then repeat the above procedure (from #3) in an iterative manner until the estimate is assumed to be the identified parameter vector.

E. Simulation Results and Discussion

The results of the parameter identification of a low-order boiler model under simulated operating conditions are listed in this section as Table 6.2 through 6.7. These results will be discussed in a narrative manner.

Six experiments were carried out by the weighted least squared regression analysis. The experiments are divided according to possible identifiable structures and number of unknown parameters (whether or not the initial conditions of state variables are known) and signal to noise ratio level. The experiments are tabulated as shown in Table 6.1.

Structure A is given by Equation 6.3 and 6.4 and Structure B is given by Equation 6.6 and 6.7. The signal to noise ratio is the ratio of the signal power of the outputs (throttle pressure, D_T and steam flow rate, W_T) to the variances of output measurement noises. The variances of the noises are 4×10^{-10} and 1×10^{-8} when the signal to noise ratio is 60 db.

If the performance index $J(\underline{z}(0))$ given in Equation 2.7 is rewritten, then

$$J(\underline{z}(0)) = \| \underline{Y}(N) - f(z(0)) \|_W^2$$

The weighting matrix W is chosen as the inverse of the covariance of the noise introduced into the output measurements. The performance index is normalized for convenience and denoted as $J(\underline{z}(0))^*$.

As mentioned earlier, the initial conditions of the state variables can be assumed to be known since a low-order model is the representation of the boiler dynamics of a system which is initially at rest and then followed by a disturbance. But the output measurements are corrupted by a noise, and the initial conditions are not known exactly. Both cases are studied in Experiment I and II for the Structure A and Experiment III and IV for the Structure B.

The convergence rate for normalized performance index $J(\underline{z})^*$ of experiments II and III are compared and are shown in Figure 6.3, and $J(\underline{z})^*$ of experiments V and VI are shown in Figure 6.4.

Comparing the results of the experiments of structure A with those of the experiments of structure B, there is no significant difference in the convergence behavior of the identification algorithm between the two different structures. But the identified parameters of the structure B are easier to relate to the parameters of the Laubli-Fenton model than those of the structure A.

In summary, an algorithm has been developed for the parameter identification of dynamic systems. It utilizes the best features

of the Gauss-Newton method, the Newton-Raphson method and the modified Gauss-Newton method. The application of the above algorithm has been made to the two identifiable canonical forms which are parametrized from the Laubli-Fenton low-order boiler model.

Table 6.1. List of experiments

Exp.	Structure	No. of parameter	S/N	Results
I	A	13(unknown I.C.)	60 db	Table 6.2
II	A	9(known I.C.)	60 db	Table 6.3
III	A	9(known I.C.)	40 db	Table 6.4
IV	B	12(unknown I.C.)	60 db	Table 6.5
V	B	8(known I.C.)	60 db	Table 6.6
VI	B	8(known I.C.)	40 db	Table 6.7

Table 6.2. Estimated parameter of Experiment I

(Structure A
S/N = 60 db
Unknown initial condition)

Parameter/Iteration	Initial	Identified
$J(z)^*$	0.10000D 01	0.1776044D-05
z_1	0.0	-0.4867316D-04
z_2	0.0	0.1362532D-04
z_3	0.0	-0.7407808D-06
z_4	0.0	0.2085594D-07
z_5	0.2717391D-06	0.3631789D-06
z_6	0.5842391D-04	0.5055734D-04
z_7	0.3994565D-02	0.3411940D-02
z_8	0.1086957D 00	0.9230709D-01
z_9	0.8695652D-02	0.1015535D-01
z_{10}	0.8700000D 00	0.9862895D 00
z_{11}	0.2717391D-06	0.2472271D-06
z_{12}	0.1300000D 00	0.5910328D-03
z_{13}	0.8700000D 00	0.9992989D 00

Table 6.3. Estimated parameter of Experiment II

(Structure A S/N = 60 db Known initial condition) Parameter/Iteration	0 Initial	5 Identified
$J(\underline{z})^*$	0.100000D 01	0.6077167D-05
z_5	0.271739D-06	0.6415899D-07
z_6	0.5842391D-04	0.4900899D-04
z_7	0.3994565D-02	0.3392798D-02
z_8	0.1086957D 00	0.9327155D-01
z_9	0.8695652D-02	0.1015337D-01
z_{10}	0.8700000D 00	0.1033935D 01
z_{11}	0.2717391D-06	0.2848169D-06
z_{12}	0.1300000D 00	0.5979398D-03
z_{13}	0.8700000D 00	0.9993104D 00

Table 6.4. Estimated parameter of Experiment III

(Structure A
S/N = 40 db
Known initial condition)

Parameter/Iteration	0 Initial	5 Identified
$J(\underline{z})^*$	0.100000D 01	0.49703D-03
z_5	0.2717391D-06	0.8163276D-07
z_6	0.5842391D-04	0.5053236D-04
z_7	0.3994565D-02	0.3458786D-02
z_8	0.1086957D 00	0.9679339D-01
z_9	0.8695652D-02	0.1030706D-01
z_{10}	0.8700000D 00	0.1017804D 01
z_{11}	0.2717391D-06	0.2908795D-06
z_{12}	0.1300000D 00	0.9049269D-05
z_{13}	0.8700000D 00	0.9990934D 00

Table 6.5. Estimated parameter of Experiment IV

(Structure B
S/N = 60 db
Unknown initial condition)

Parameter/Iteration	0 Initial	5 Identified
$J(z)^*$	0.100000D 01	0.5784069D-05
z_1	0.0	0.9920448D-05
z_2	0.0	0.1499598D-03
z_3	0.0	0.6052366D-03
z_4	0.0	0.5100734D-04
z_5	0.8695652D-02	0.8779424D-02
z_6	0.8695652D-02	0.6867185D-02
z_7	0.3125000D-04	0.3385793D-04
z_8	0.3125000D-02	0.2137950D-02
z_9	0.1000000D 00	0.8346474D-01
z_{10}	0.7565217D-02	0.1054123D-01
z_{11}	0.1300000D 00	0.8018787D-03
z_{12}	0.8700000D 00	0.9991065D 00

Table 6.6. Estimated parameter of Experiment V

(Structure B S/N = 60 db Known initial condition) Parameter/Iteration	0 Initial	4 Identified
$J(\underline{z})^*$	0.100000D 01	0.6937482D-05
z_5	0.8695652D-02	0.1012139D-01
z_6	0.8695652D-02	0.8524324D-02
z_7	0.3125000D-04	0.2480657D-04
z_8	0.3125000D-02	0.2597521D-02
z_9	0.1000000D 00	0.8478637D-01
z_{10}	0.7565217D-02	0.1049438D-01
z_{11}	0.1300000D 00	0.6509907D-03
z_{12}	0.8700000D 00	0.9993141D 00

Table 6.7. Estimated parameter of Experiment VI

(Structure B
S/N = 40 db
Known initial condition)

Parameter/Iteration	0 Initial	4 Identified
$J(\underline{z})^*$	0.100000D 01	0.4707073D-03
z_5	0.8695652D-02	0.1013411D-01
z_6	0.8695652D-02	0.8520751D-02
z_7	0.3125000D-04	0.2455783D-04
z_8	0.3125000D-02	0.2601675D-02
z_9	0.1000000D 00	0.8440967D-01
z_{10}	0.7565217D-02	0.1051035D-03
z_{11}	0.1300000D 00	0.4861681D-03
z_{12}	0.8700000D 00	0.1000469D 01

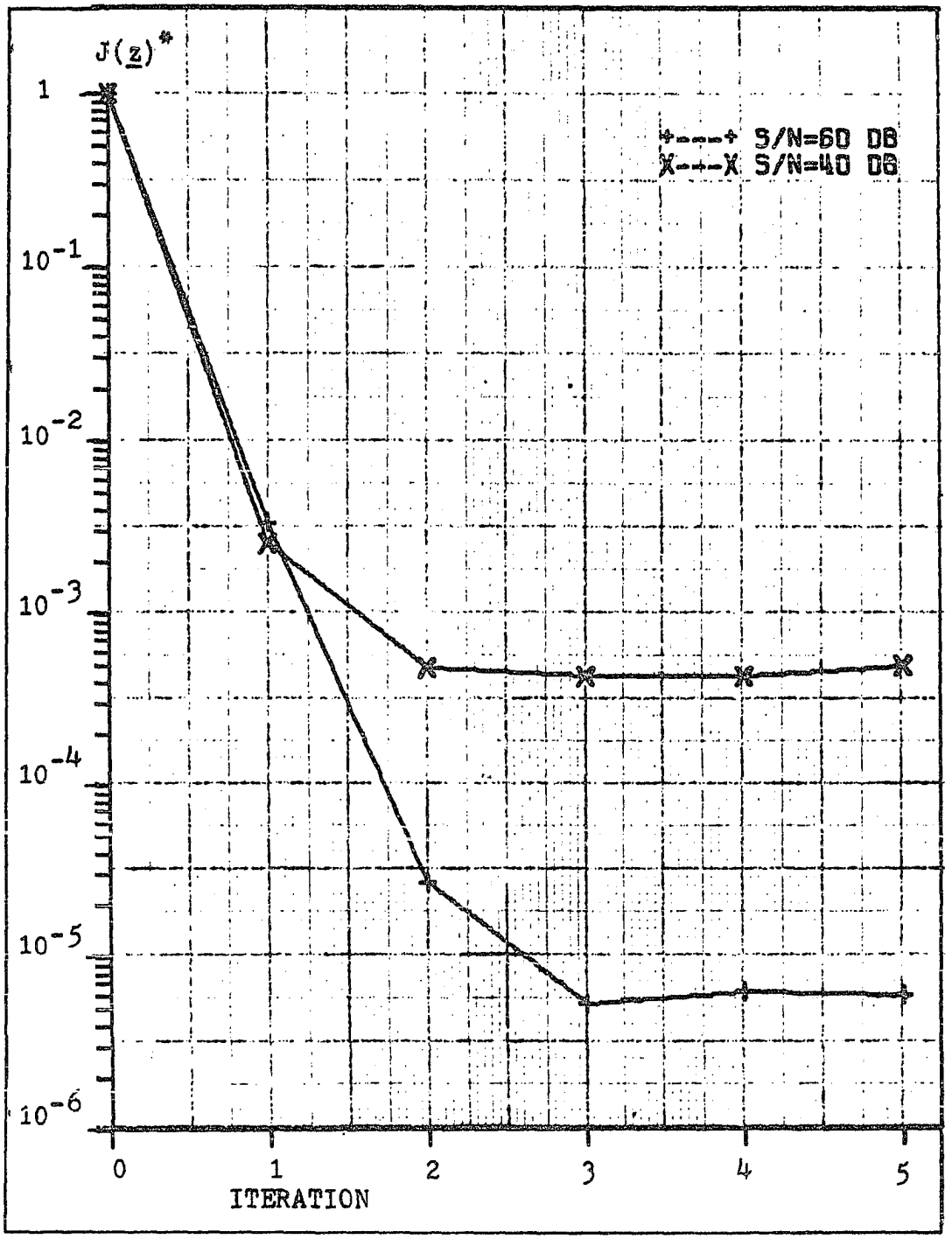


Figure 6.3. Convergence rate for normalized performance index $J(\underline{z})^*$ of structure A

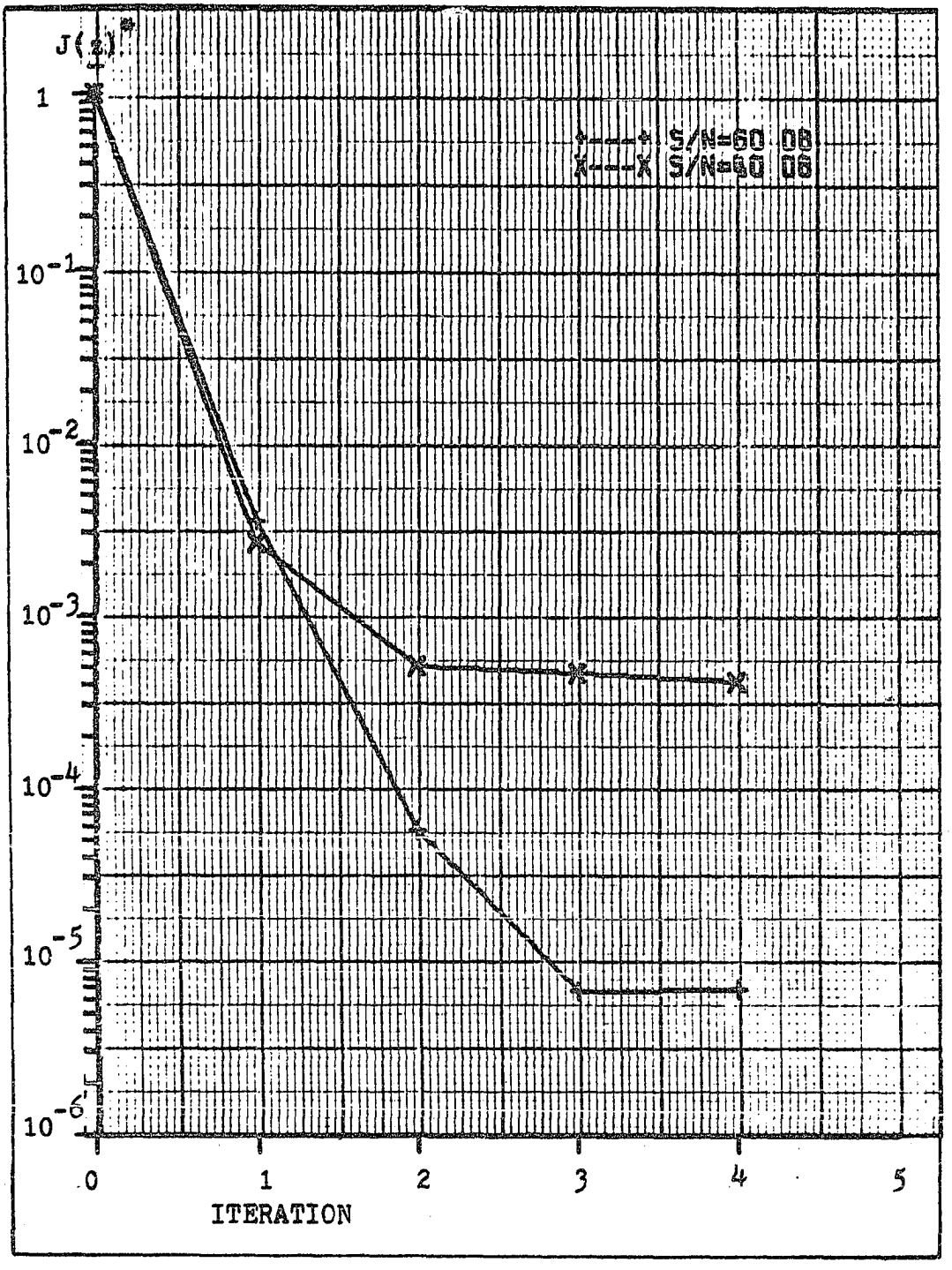


Figure 6.4. Convergence rate for normalized performance index J(z) of structure B

VII. FIELD TEST PROCEDURES OF BOILER DYNAMICS

Extensive simulation studies of the parameter identification of low-order boiler models were made with the identification algorithm in the previous chapter. A very important part of the parameter identification of a process dynamic model from input/output data is to get meaningful data from field tests. In this chapter, we are going to develop these procedures in order to get boiler dynamic response data. The test data are intended for low-order boiler dynamic modeling which might be used for power system stability studies.

A. Power Plant Design Data

The field tests were performed at the Iowa Public Service George Neal Station, Unit #2, Sioux City, Iowa, on December 24 and 30, 1974. The following is a list of design data:

1. Turbine Generator:

   320, 576 KW gross at 2400 Psig,

   1000^{o}F/1000^{o}F Reheat

2. Steam Generator:

   2,320,000 Lb/Hr Main Steam Flow

   Superheater Outlet 2625 Psig/1005^{o}F

   2,030,000 Lb/Hr Reheat Flow

   Boiler Efficiency 87.9%

B. Field Test Procedures

Three different types of tests were performed at two different load levels (about 90% of full load and 70% of full load). The tests are boiler energy storage test, boiler thermal inertia test and multiple-input/multiple-output test, and they are listed in Table 7.1.

Table 7.1. List of Tests

Load level	Type of tests	Generation	Throttle pressure
90%	Energy storage	285 MW	2408 psi
	Thermal inertia	260 MW	2448 psi
	Multiple input/ output test I	290 MW	2368 psi
	Multiple input/ output test II	305 MW	2406 psi
70%	Energy storage	180 MW	2529 psi
	Thermal inertia	220 MW	2378 psi
	Multiple input/ output	220 MW	2390 psi

As a general test condition, all the variables of interest should be in the steady-state condition before initiating the test. This implies that the boiler inputs (fuel, air, feedwater) and the boiler outputs (superheater steam temperature, throttle pressure, and steam flow rate) should be maintained at constant values at their setpoints for a sustained period of time before the test, say 5 to 10 minutes.

1. Boiler energy storage test

An important feature of a thermal system is the ability of the system to store energy. This stored energy is available to supply a sudden increase in load demand up to some limiting value. Following a sudden increase in load, stored energy is released from several sources. Among them, the stored thermal energy is released from the boilers and is of substantial quantity. The energy stored in boilers also varies greatly with boiler design. Drum type boilers have a great deal more metal and fluid than once through boilers.

This test is intended for examining the energy storage behavior by applying a sudden load change while the inputs to the boiler, such as fuel, air and feedwater, remain at constant level.

2. Boiler thermal inertia test

The thermal inertia represents the time required to bring the boiler energy of steel and fluid to new steady state values following a change in heat supply. The actual characteristics of steam generation, such as steam flow to the turbine and throttle pressure, vary widely depending upon the type of fuel (coal, gas and oil) and the method of fuel preparation and feeding.

The definition of the thermal inertia in this dissertation represents the time required to bring the throttle pressure and the steam flow rate to new steady state conditions following a change in coal flow rate.

This test is intended for examining the dynamic response of the boiler outputs, steam flow rate to the turbine and throttle pressure, by applying a sudden boiler input change while the load demand signal remains at a constant level. The sudden boiler input change can be implemented by changing the throttle pressure set-point adjustment knob from one value to another, say from 2400 psi to 2450 psi. The fuel demand signal is biased by throttle pressure error signal which is generated in a relay by comparing throttle pressure with the setpoint selected by the operator (manually). In order to keep fuel and air within the safe combustion limit, the air control loop should be put on the automatic control mode. The feedwater must be on automatic to maintain drum water level. The drum level control should be tightly tuned.

3. Multiple input/multiple output test

The previous two tests are done by changing one control input at a time, i.e., while the other control inputs are held at constant level. It is more desirable to have a test where all the inputs are perturbed simultaneously since the boiler-turbine unit is a typical multiple-input/multiple-output system.

This test is intended for examining the boiler output response by allowing all the control inputs to change upon a sudden load demand signal.

Consider the sequence of events occurring upon a sudden load increase from normal operating conditions where all the boiler-turbine

variables are at steady state and all the control inputs are at automatic control mode. The control valve is opened upon the load demand signal instantly. Then, first stage pressure increases and throttle pressure decreases until the firing rate has been increased sufficiently to provide not only the energy required at the new load level, but also that required to replace the energy (steam flow) borrowed during the load change. By providing a transient over-firing, the system rapidly moves the fuel, air, feedwater to a value that provides the required steam flow at the correct throttle pressure. Therefore, by changing the load demand signal, we can disturb all the control variables within the maximum allowable deviations of the boiler variables at the normal operating conditions.

Load demand signal changes in a up-down-up-down pattern were made with appropriate time intervals in order to get good average test data.

C. Measurements and Recordings

Fourteen variables were recorded on a multi-channel FM magnetic recording machine, and later they are digitized for use on a digital computer.

Some of the recorded variables are

1. load demand signal,
2. total control valve flow signal,
3. total control valve position signal,
4. total coal flow rate,

5. throttle pressure,

6. first stage pressure (turbine), and

7. steam flow rate to turbine.

All of these variables were available as calibrated signals from electrical circuit terminal points in the electronic-type boiler control systems (BACC 820) and in the turbine control systems (EHC).

Special precautions were taken on the signal of the Electro-hydraulic Control System (EHC) of the turbine. Fuel input to the boiler is regulated by adjusting the speed of coal feeders and measured feeder speeds are summed, in a relay, with ignition gas flow to produce a total fuel flow signal.

The signals were digitized, the sampling rate of throttle pressure and steam flow rate were 10 seconds each and the sampling rate of the other signals is 1 second.

VIII. EXPERIMENTAL RESULTS AND DISCUSSION

In the previous chapter, the field test procedures of boiler dynamics were described. The results of the parameter identification of low-order boiler model with the actual field test data will be presented in this chapter and will be discussed in a narrative manner.

A. Boiler Energy Storage Low-order Model

For a constant firing rate, C_F, an increase in the control valve area, A, increases the steam flow rate to the turbine and decreases throttle pressure. The increase in turbine power, which is directly proportional to the steam flow rate, comes from the boiler stored energy.

Considering the above facts, it is assumed that the boiler storage behavior can be represented by two first-order transfer functions as shown in Figure 8.1.

The control valve signal is measured in percentages of maximum values. There is a non-linear relationship between the control valve position and the control valve opening area to which the steam flow rate to the turbine is directly proportional. The control valve system is designed such that the steam flow rate to the turbine, W_T, is directly proportional to the control valve flow signal, C_V. The control valve system has been shown in Figure 5.5. Since the valve opening area is not directly measureable in the field tests,

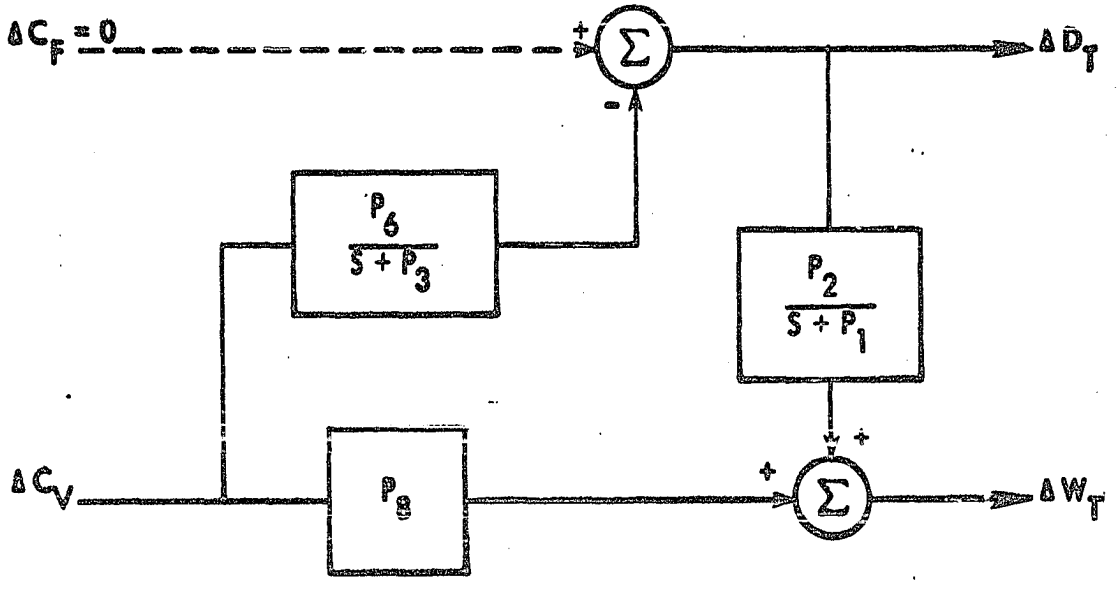


Figure 8.1. Block diagram of boiler energy storage model

the control valve flow signal, C_V, is used for the input to the assumed boiler energy storage model.

If the model is represented by a state variable form then

$$\frac{d}{dt}\underline{x} = A\underline{x} + B\underline{u}$$

$$\underline{y} = C\underline{x} + D\underline{u}$$

where $A = \begin{bmatrix} -P_1 & -P_2 \\ 0 & -P_3 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ P_6 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $D = \begin{bmatrix} P_8 \\ 0 \end{bmatrix}$

The output vector $\underline{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ where y_1 is the steam flow rate to the turbine ($= \triangle W_T$), and y_2 is the throttle pressure ($= \triangle D_T$). The state vector is $\underline{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. The input vector $\underline{u} = u_1$ where u_1 is the control valve flow signal ($= \triangle C_V$).

The parametrization of this linear model can be shown to be globally identifiable from the transfer function.

The identified parameters of the boiler energy storage model are shown in Table 8.1. The parameters P_1 and P_2 are associated with the lag between the throttle pressure and the steam flow rate and do not change much with respect to the load levels. The parameters P_3 and P_4 are associated with the energy storage in the boiler drum and

superheaters, and the time constant at the load level 180MW is shown to be longer than at the load level 285MW. The difference in the time constant values could be explained partially by the non-linearity of the boiler energy storage process.

Table 8.1. Identified parameter values of the boiler energy storage model

Parameters	Load level	
	285 MW	180 MW
P_1	0.62983D-02	0.54670D-02
P_2	0.72914D-02	0.68810D-02
P_3	0.28918D-02	0.79215D-03
P_6	0.45908D-02	0.13875D-02
P_8	0.82575D 00	0.36472D 00

The responses of the identified boiler energy storage model and the field test data are shown in Figure 8.2 and 8.3. The solid line (——) represents model responses and the symbol (+++++) represents measured data.

B. Thermal Inertia Low-order Model

The actual firing system characteristics of power plants vary widely, depending upon the type of fuel and method of fuel preparation and feeding. Also, the steam generation depends upon the type of boiler circulation. Usually oil or gas firing systems have a rapid response to fuel input change compared to coal firing systems.

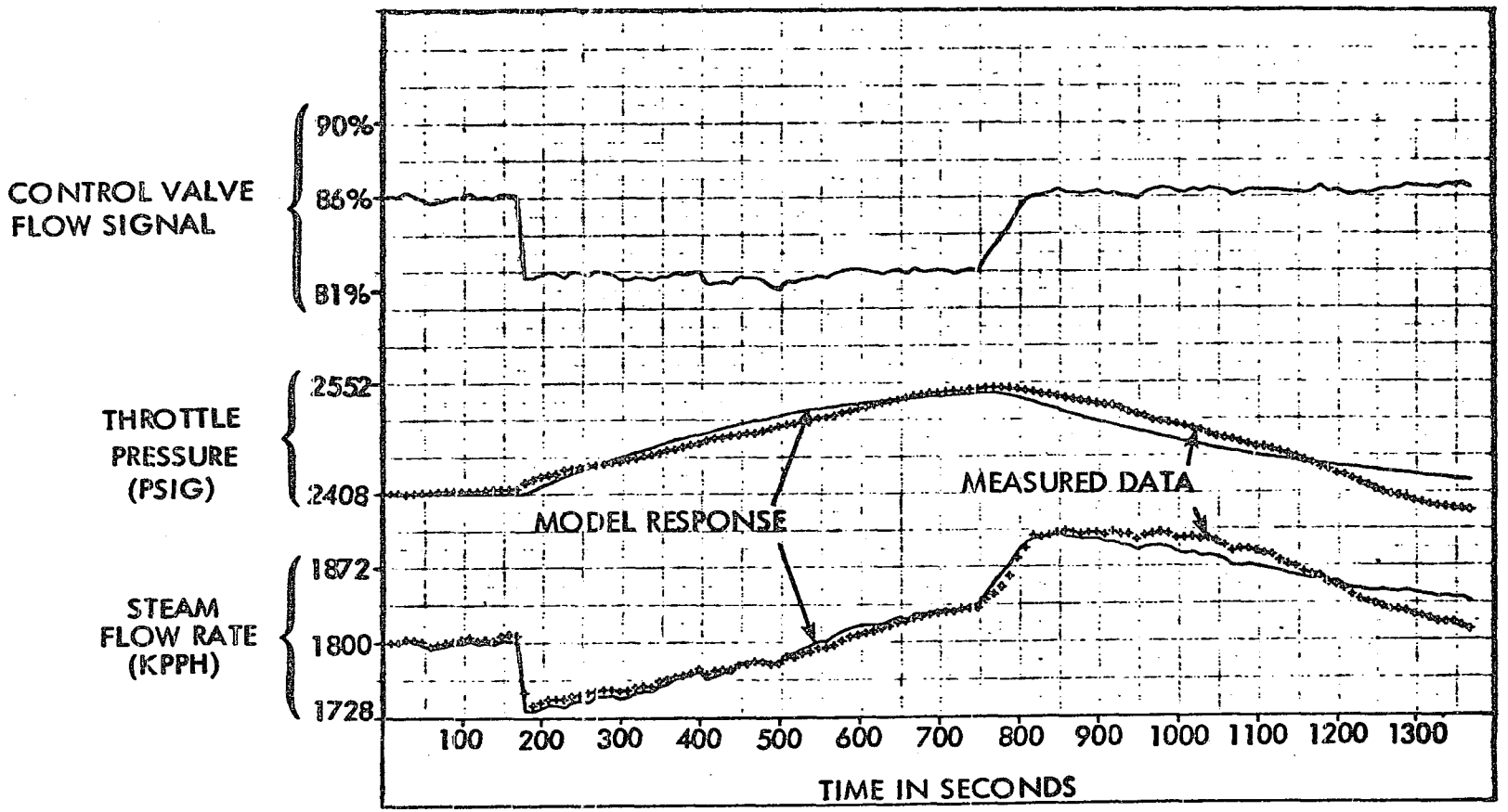


Figure 8.2. Comparison of the measured data and the responses of the boiler energy storage model at load level 285 MW

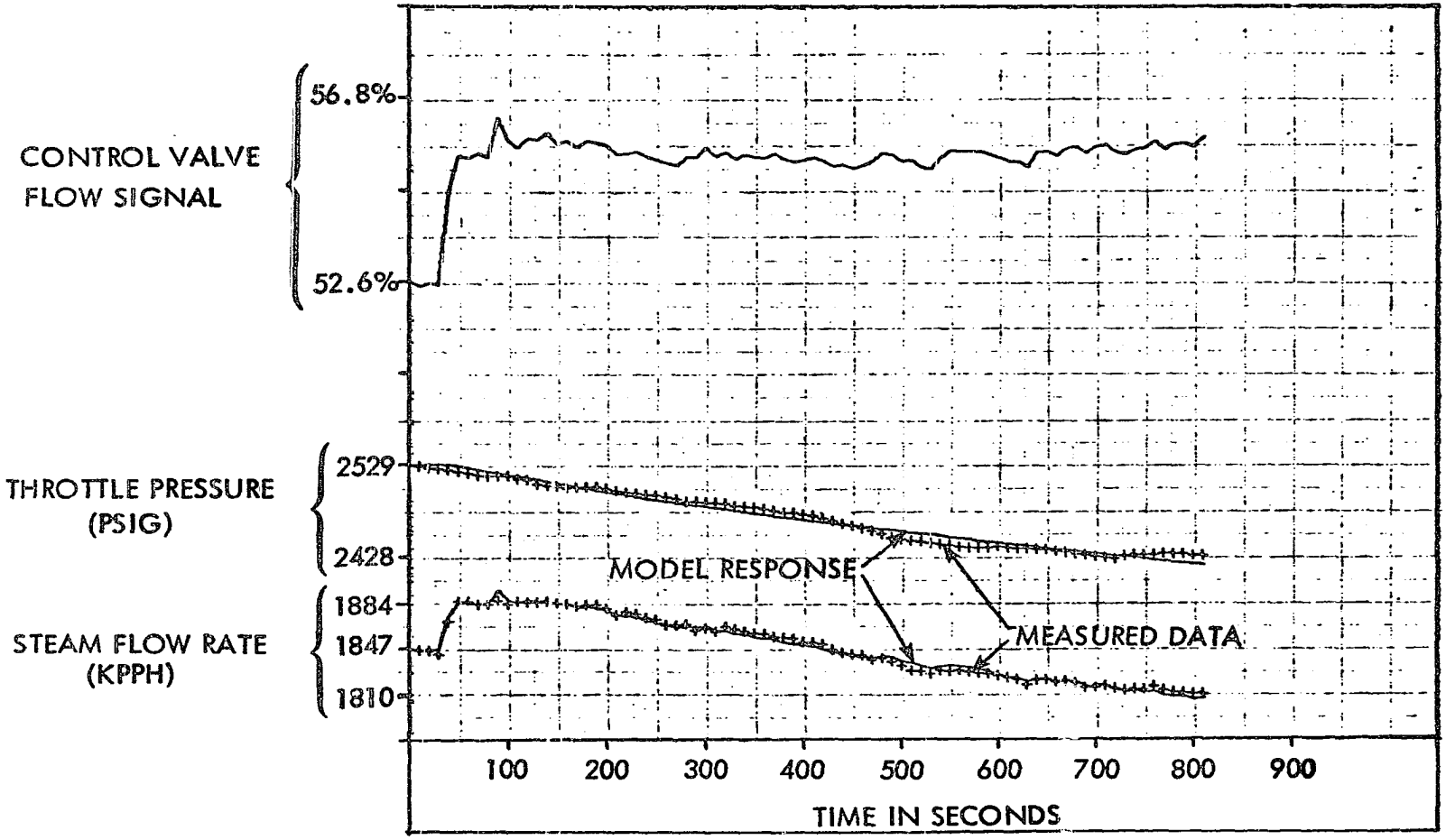


Figure 8.3. Comparison of the measured data and the responses of the boiler energy storage model at load level 180 MW

The lag between the coal flow rate to the boiler and the throttle pressure is assumed to be represented by a second order transfer function and the lag between the throttle pressure and the steam flow rate is represented by a first order transfer function in a similar way which was shown in the boiler energy model. The structure of the thermal inertia model is shown in Figure 8.4.

The structure of the model is quite similar to that of the Laubli-Fenton model (1, 2, 32); however, it differs in the fact that a first-order time lag appears between the throttle pressure (ΔD_T) and the steam flow rate (ΔW_T). It is customary in most models appearing in the literature to assume there is no lag between throttle pressure and steam flow rate. Referring to Figures 8.5 and 8.6, it is difficult to notice any time lag between these two quantities. However, superimposing the actual measured data does reveal a lag as shown in Figures 8.7 and 8.8. This effect appears to be due to small variations in temperature which are usually ignored.

If the thermal inertia model is represented by a state variable form, then

$$\frac{d}{dt}\ \underline{x} = A\underline{x} + B\underline{u}$$

$$\underline{y} = C\underline{x} + D\underline{u}$$

where

$$A = \begin{bmatrix} -P_1 & +P_2 & 0 \\ 0 & 0 & 1 \\ 0 & -P_4 & -P_5 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ P_7 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \end{bmatrix}$$

The output vector $\underline{y} = (y_1, y_2)^T$ where y_1 is the steam flow rate to the turbine ($= \Delta W_T$), and y_2 is the throttle pressure ($= \Delta D_T$). The state vector is $\underline{x} = (x_1, x_2, x_3)^T$. The input vector $\underline{u} = u_1$ where u_1 is the coal flow rate to the boiler ($= \Delta C_F$).

The five parameters, P_1, P_2, P_4, P_5 and P_7, are to be identified and the parametrization of the thermal inertia model can be shown to be globally identifiable from the transfer function. The identified parameters of the thermal inertia model are shown in Table 8.2. The parameters, P_1 and P_2, are associated with the lag between the throttle pressure and the steam flow rate and their values at 260 MW load level are within small variation respect to the corresponding parameter values of the boiler energy storage model. The results show that the thermal inertia responses are slower than at 220 MW. The responses of the identified thermal inertia model and the field test data are shown in Figures 8.4 and 8.5.

Table 8.2. Identified parameter values of the thermal inertial model

Parameters	Load level	
	260 MW	220 MW
P_1	0.63596D-02	0.18002D-01
P_2	0.74843D-02	0.16777D-01
P_4	0.11567D-04	0.73242D-04
P_5	0.7000 D-02	0.28980D-01
P_7	0.19482D-04	0.63266D-04

C. Multiple-Input/Multiple-Output Low-order Boiler Model

The structure of a low-order boiler model is based on the physical consideration and the results of the boiler energy storage modeling and the thermal inertia modeling. It was decided to use of fourth order low-order model of the boiler. The inputs to the model are the coal flow rate ($= \Delta C_F$), and the control valve flow signal ($= \Delta C_V$) assuming that the effective valve opening area is directly proportional to the control valve flow signal. The outputs of the model are the throttle pressure ($= \Delta D_T$), and the steam flow rate ($= \Delta W_T$). The model structure is shown in Figure 8.9. Again, the model is the same as that of Laubli and Fenton with the exception of the first order time lag appearing between throttle pressure (ΔD_T) and steam flow rate (ΔW_T).

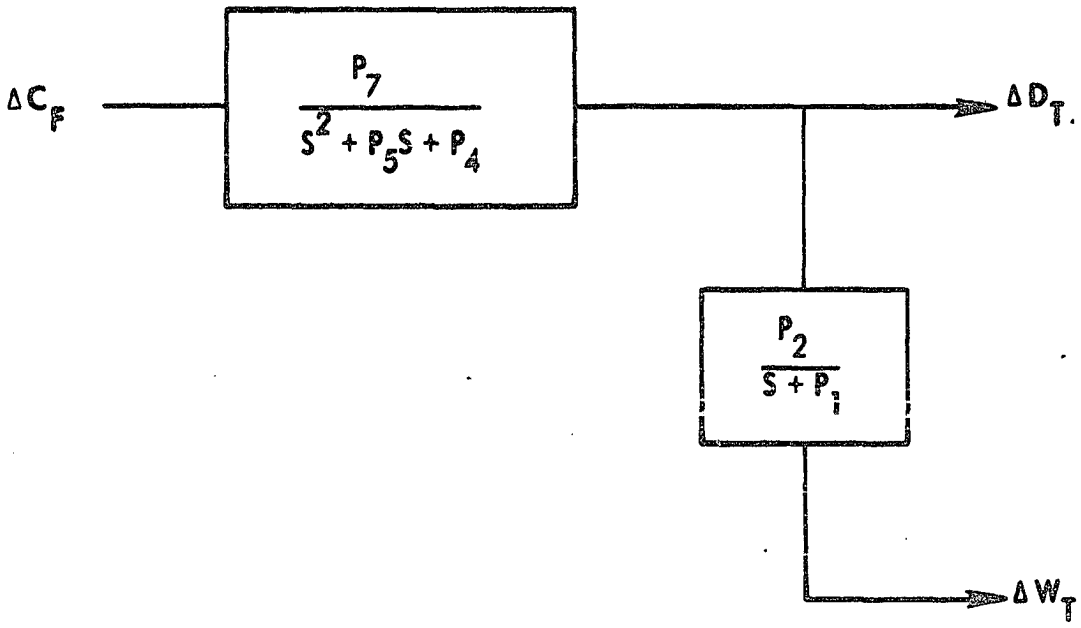


Figure 8.4. Block diagram of thermal inertia model

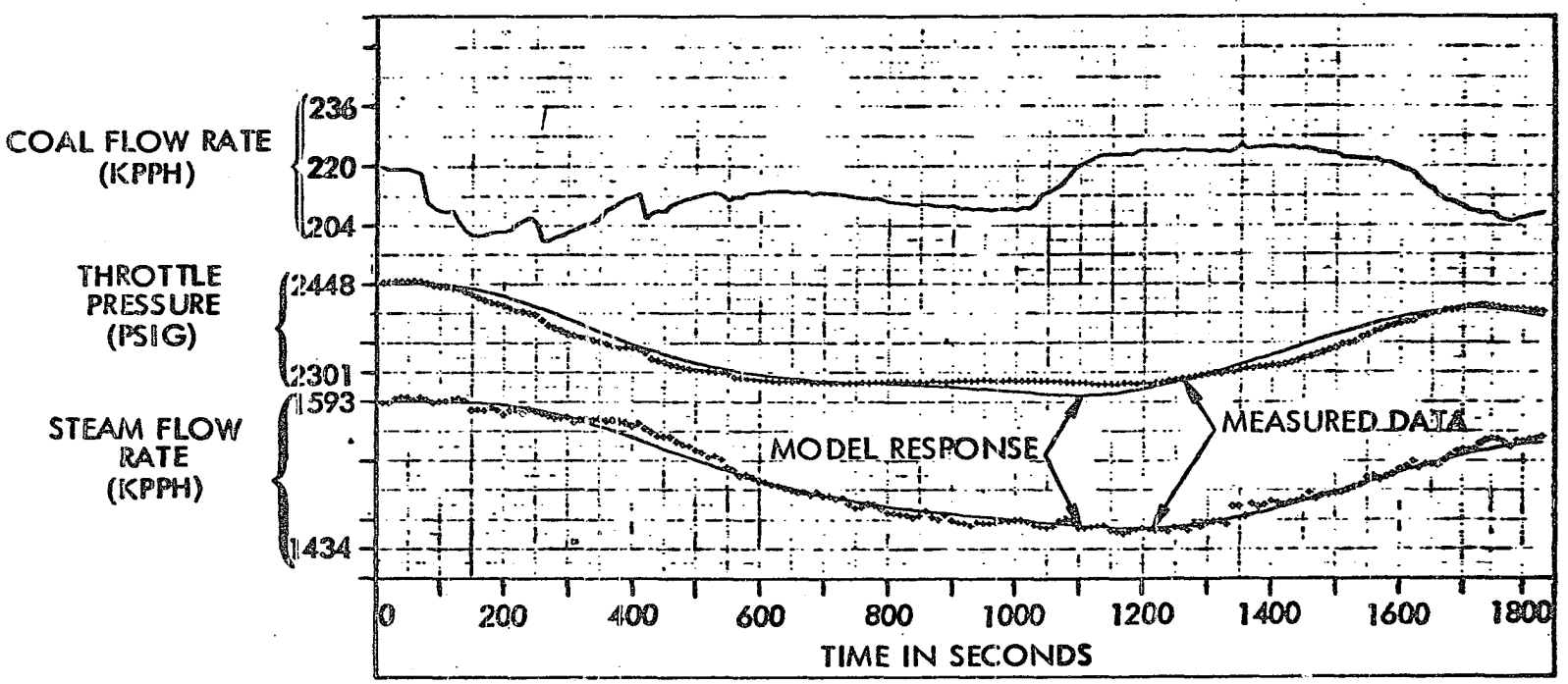


Figure 8.5. Comparison of the measured data and the responses of the thermal inertia model at load level 260 MW

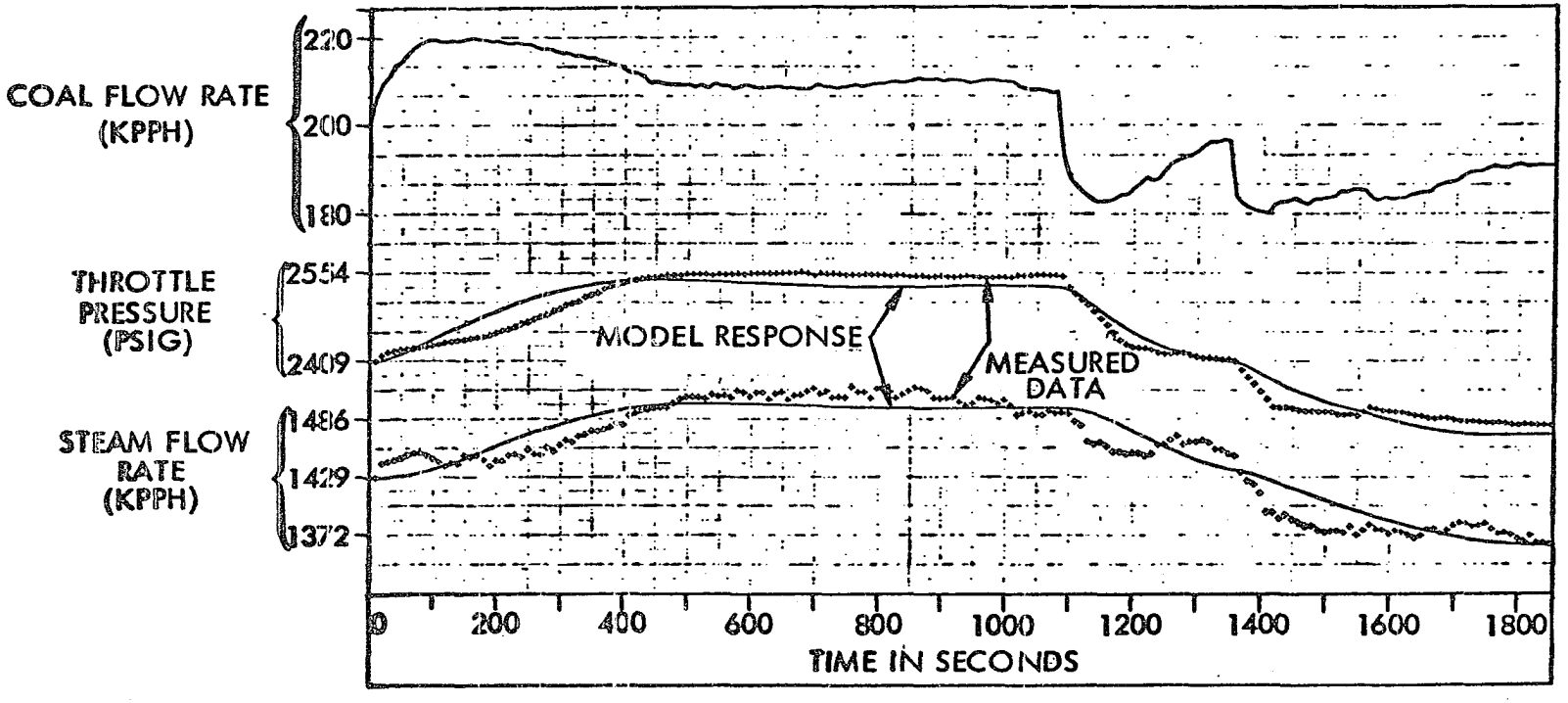


Figure 8.6. Comparison of the measured data and the responses of the thermal inertia model at load level 220 MW

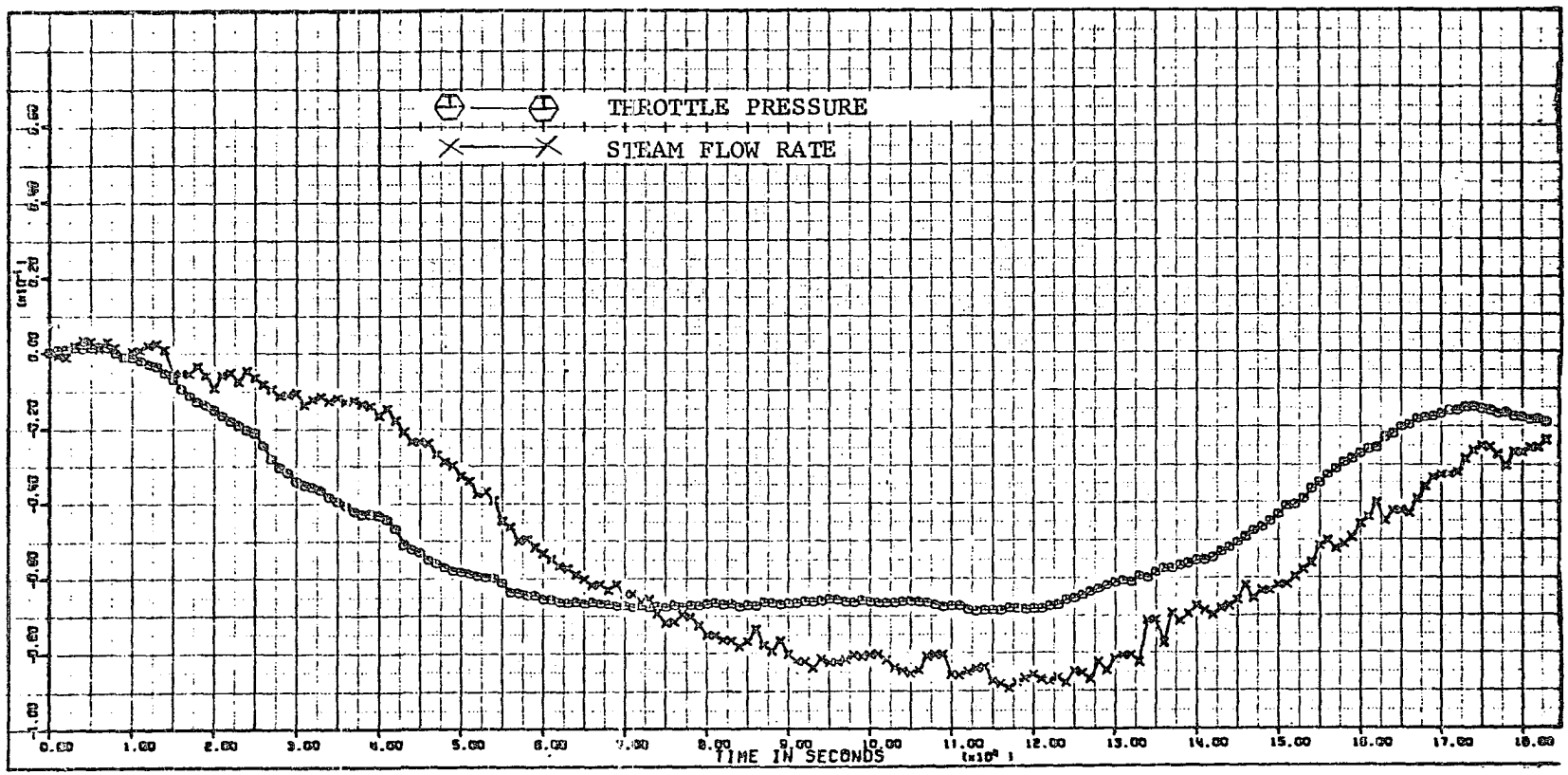


Figure 8.7. Lag between measured throttle pressure data and measured steam flow rate data of thermal inertia test at 260 MW

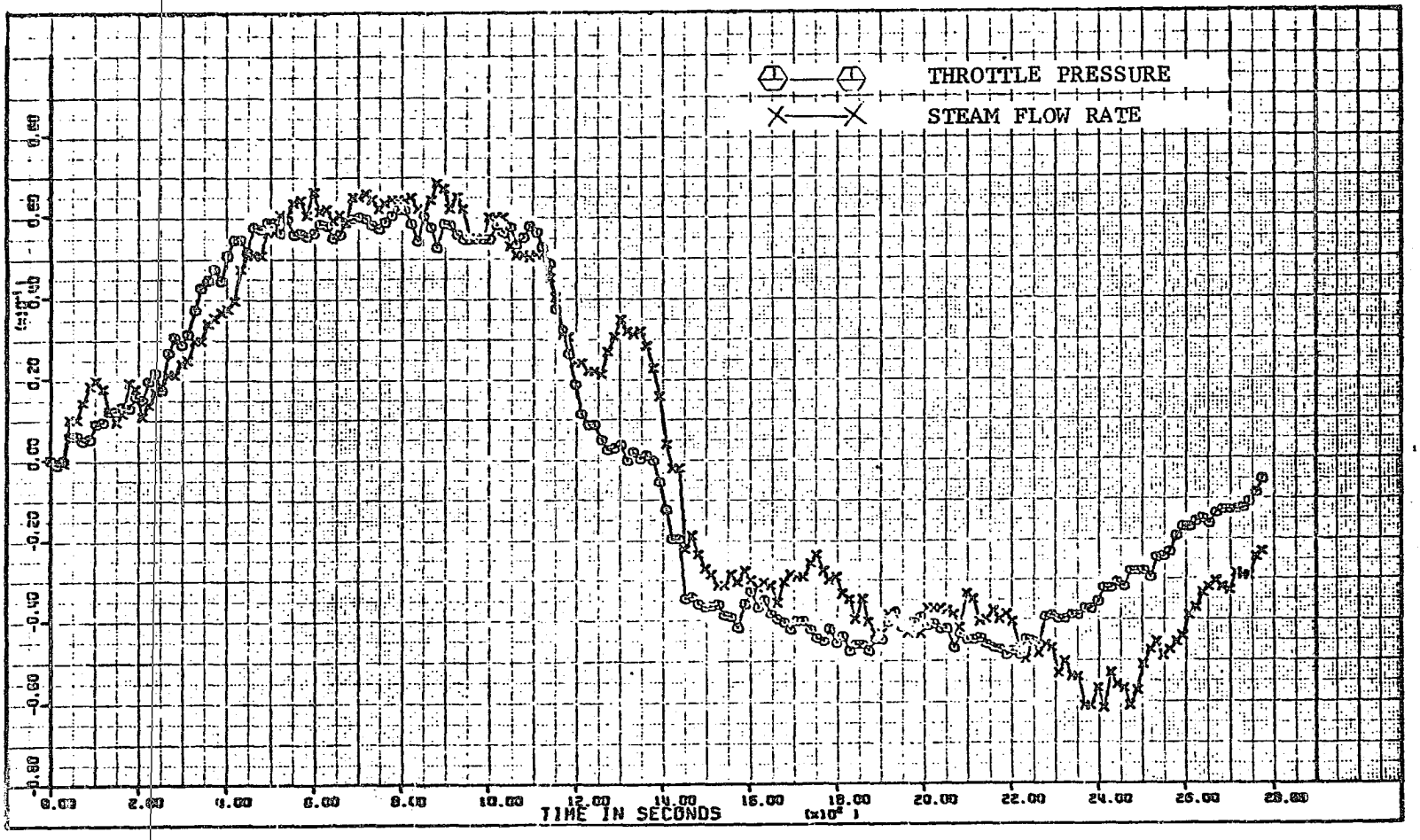


Figure 8.8. Lag between measured throttle pressure data and measured steam flow rate data of thermal inertial test at 220 MW

If the model is represented by state variable form then

$$\frac{d}{dt}\underline{x} = A\underline{x} + B\underline{u}$$

$$\underline{y} = C\underline{x} + D\underline{u}$$

where

$$A = \begin{bmatrix} -P_1 & -P_2 & P_2 & 0 \\ 0 & -P_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -P_4 & -P_5 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ P_6 & 0 \\ 0 & 0 \\ 0 & P_7 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} P_8 & 0 \\ 0 & 0 \end{bmatrix}$$

The output vector $\underline{y} = (y_1, y_2)^T$ where y_1 is the steam flow rate ($= \triangle W_T$), and y_2 is the throttle pressure ($= \triangle D_T$). The input

vector $\underline{u} = (u_1, u_2)^T$ where u_1 is the control valve flow signal ($= \Delta C_V$), and u_2 is the coal flow rate to the boiler ($= \Delta C_F$). The state vector is $\underline{x} = (x_1, x_2, x_3, x_4)^T$.

The eight parameters, P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, are to be identified. The parametrization of the multiple-input/multiple-output low-order boiler model can be shown to be globally identifiable from the transfer function. The identified parameter values are shown in Table 8.3. The responses of the identified low-order boiler model and multiple-input/multiple output test data are shown in Figures 8.10, 8.11 and 8.12.

The results of the modeling show that the model fits the actual field test data in least square error sense. The test data of the multiple-input/multiple-output boiler test at the load level 217 MW can be used for non-linear modeling since the boiler system was disturbed too much by decreasing the load by 25 MW.

In summary, the parameter identification results of low-order boiler model from the actual field test data have been presented in this chapter. The experiments at the load level of about 90% of full load have been done extensively, and the experiments at about 70% of full load have been done to check how the results vary depending upon the load level.

Table 8.3. Identified parameter values of the multiple input/output boiler low-order model

Parameters	Load level		
	290 MW	305 MW	217 MW
p_1	0.52580D-02	0.12916D-02	0.12053D-02
p_2	0.76233D-02	0.88963D-02	0.64325D-03
p_3	0.37800D-02	0.41391D-02	0.71617D-02
p_4	0.18402D-04	0.23031D-04	0.80212D-04
p_5	0.42255D-02	0.84701D-02	0.43964D-02
p_6	0.25490D-02	0.60926D-02	0.13414D-02
p_7	0.639087D-05	0.154405D-04	0.32241D-05
p_8	0.11536D+01	0.16527D 01	0.12384D 01

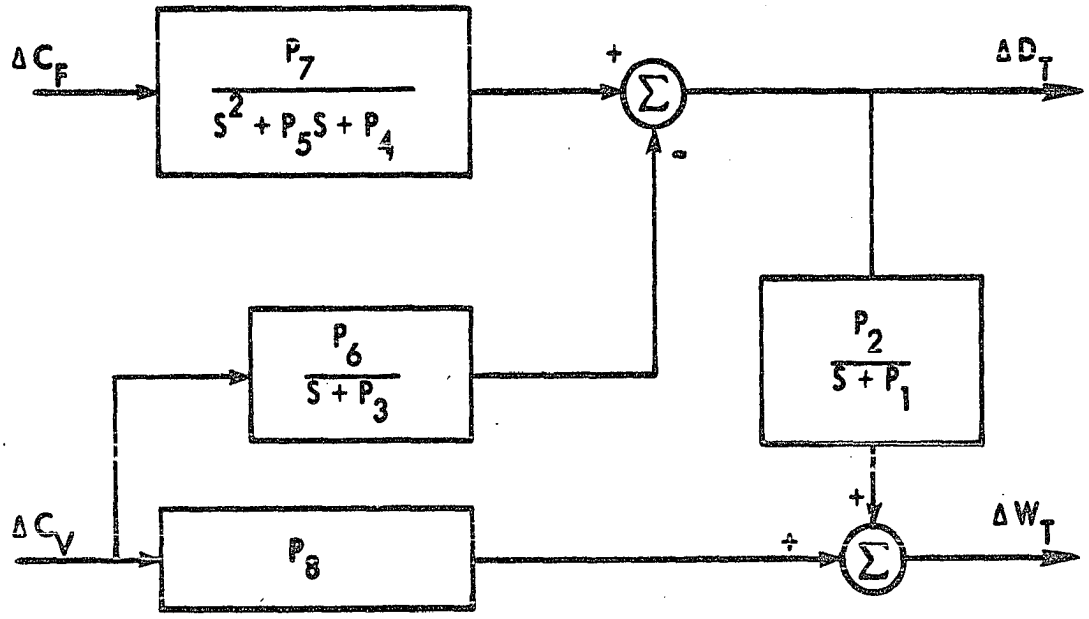


Figure 8.9. Block diagram of multiple input/output low-order boiler model

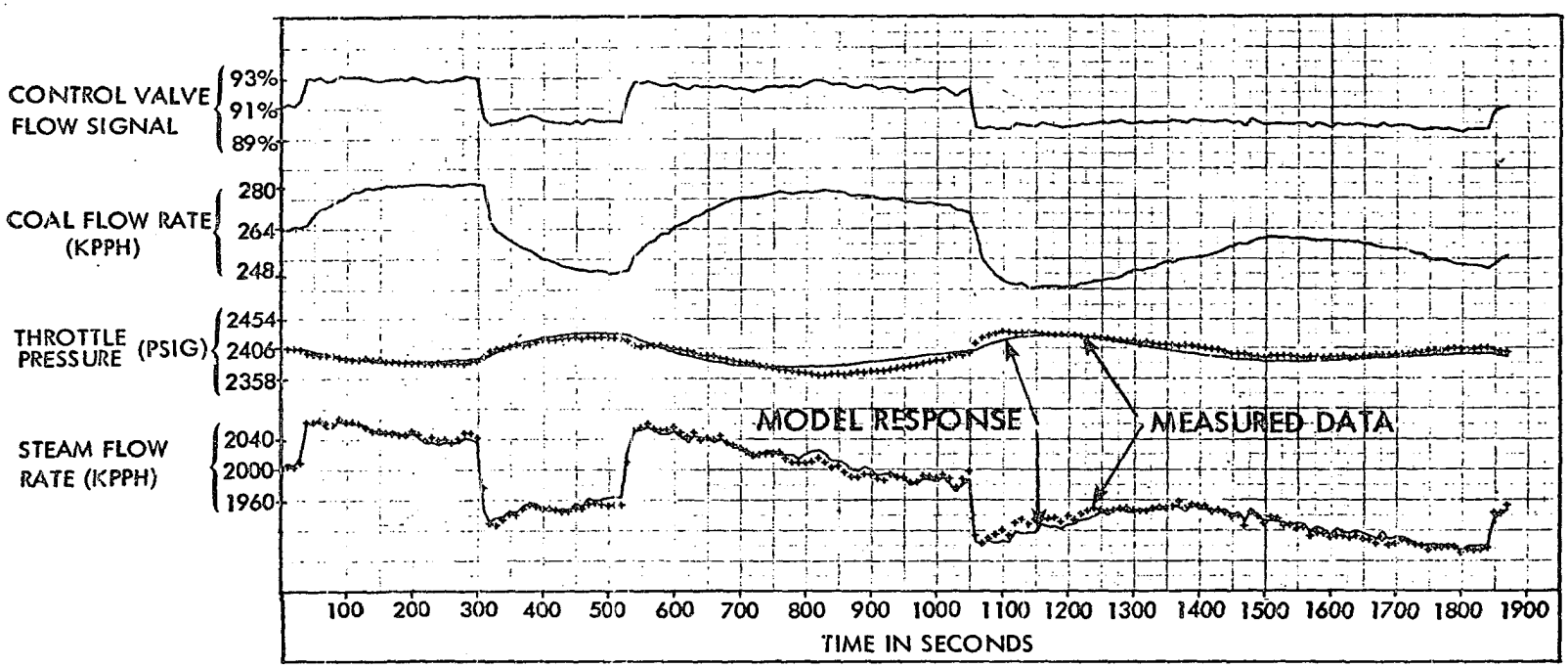


Figure 8.10. Comparison of the measured data and the responses of the model at 305 MW in gross generation

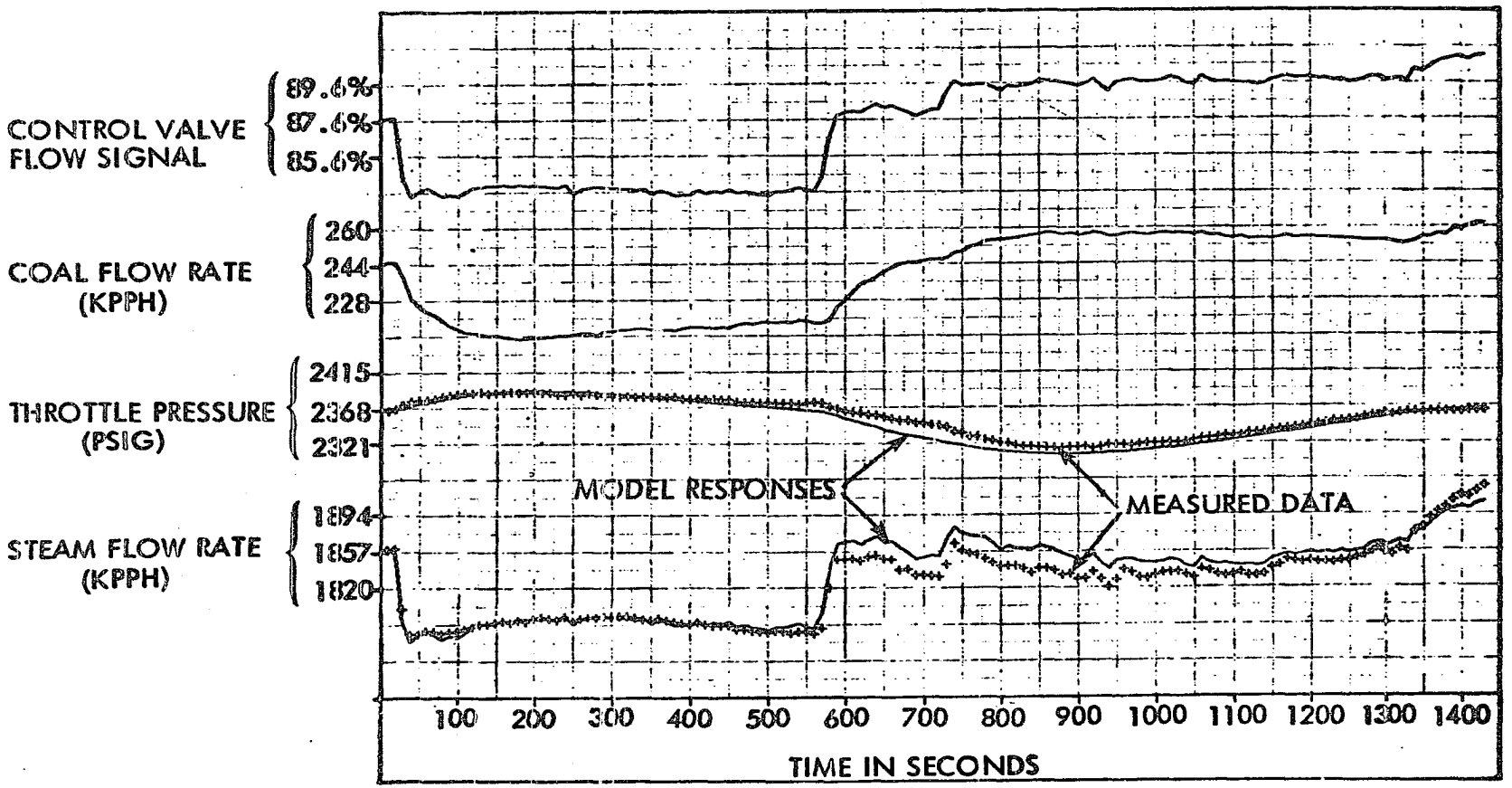


Figure 8.11. Comparison of the measured data and the responses of the model at 290 MW in gross generation

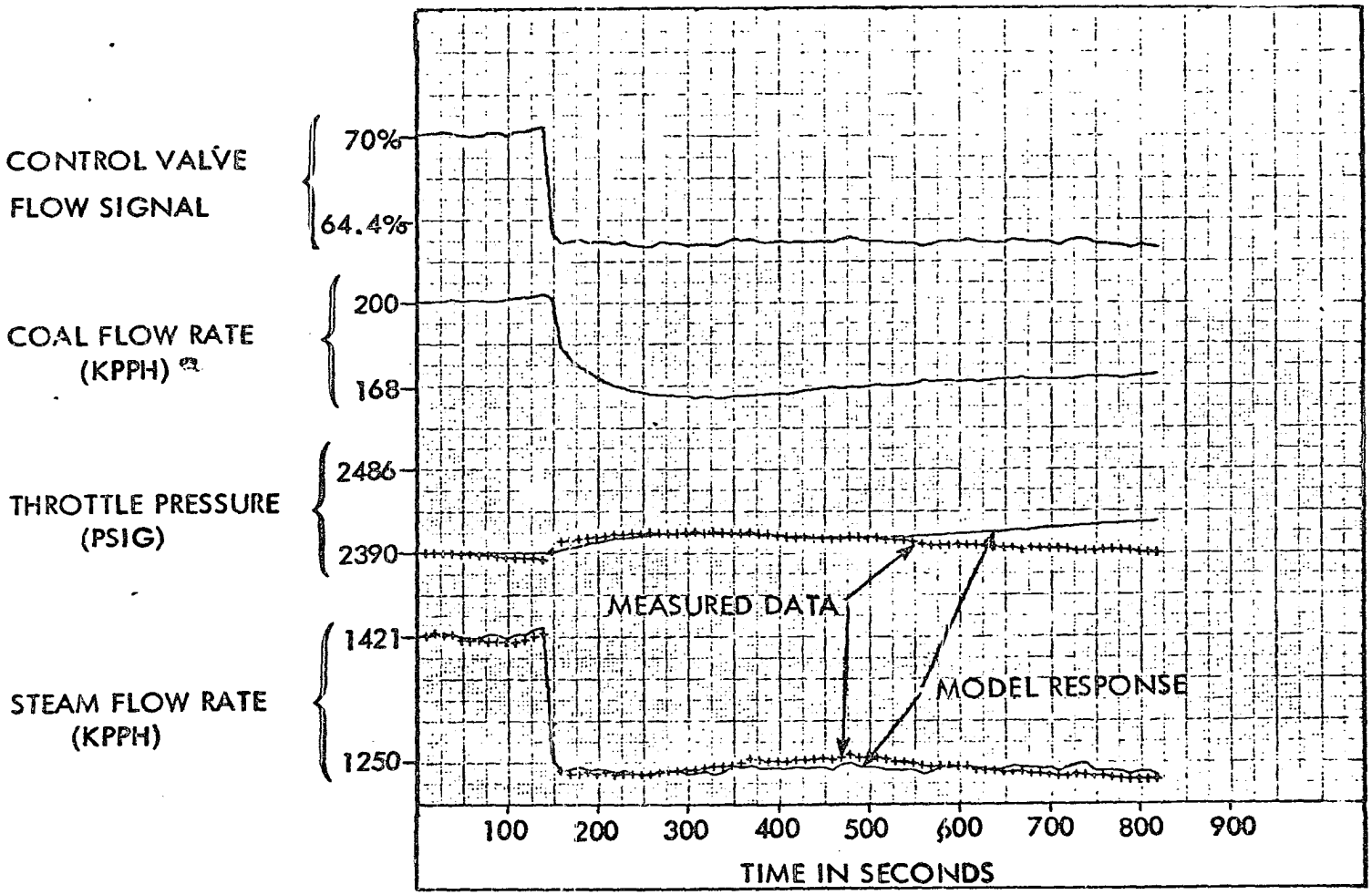


Figure 8.12. Comparison of the measured data and the response of the model at 217 MW in gross generation

IX. CONCLUSIONS

A computational procedure has been developed for application of a weighted least squares regression method to the identification of multiple-input/multiple-output continuous systems in the presence of measurement noise. The identifiability of different representation forms of linear systems is fully investigated. Canonical parametrization forms are proposed for low-order models when there is not much information available on the model structure or parameter values.

A very important part of the parameter identification of a process dynamic model is to collect meaningful data. A test procedure has been developed for the low-order boiler model.

The application of the algorithm has been made to the estimation of parameters in a low-order boiler model from the actual field test data. The experimental results show that it is practical to identify the parameters of a multiple-input/multiple-output, low-order boiler model by a least squared regression method. It is basic to the regression approach that only a finite number of response measurements be included. Thus data storage problems are avoided.

A low-order boiler model is obtained. The model has two inputs (control valve flow signal and coal flow rate) and two outputs (throttle pressure and steam flow rate to the turbine). The primary contribution to modeling is in the verification by closed loop dynamic tests with simultaneous multiple inputs from actual operating data.

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Finally, my gratitude to my wife, Hae, for her understanding and patience during these years.

XII. APPENDIX

FORTRAN PROGRAM FOR PARAMETER IDENTIFICATION

```
C         MAIN PROGRAM
C         THIS PROGRAM IS FOR THE PARAMETER IDENTIFICATION OF
C         MULTIPLE-INPUT/MULTIPLE-OUTPUT TEST AT 305 MW GENERATION.
C
C
      IMPLICIT REAL*8(A-H, O-Z)
      COMMON X(12), G(08), FF(2, 8), RA(8,8), COST
      COMMON/CONST/DELT, NOBS, NPRMT, ITRT, LIMIT, NCST
      COMMON/INPUT/U1, U2, CV(1000), CF(1000)
      COMMON/OUTPUT/DT(200), WT(200)
      DIMENSION L(8), M(8), RC(8), P(8), RD(8)
      EQUIVALENCE (X(5),P(1) )
      EQUIVALENCE(NSTATE, NPRMT)
      REAL*4 CV, CF, CH5, CH6
      EXTERNAL FUNCT
C
C
C         DATA PREPARATION
C         DELT IS THE SAMPLING TIME OF OUTPUT(WT, DT) OBSERVATIONS
C         NSETPT IS THE NUMBER OF DATA SETS OF INPUTS(CV, CF)
C
C
      NSETPT=1900
      DELT=10.0D0
      NOBS=(NSETPT-1)/DELT
      NS=(NSETPT-2)/2
      READ (5,501) (CV(I), I=1, NS)
      READ (5,501) (CF(I), I=1, NS)
      READ (5,501) (DT(I), I=1, NOBS)
      READ (5,501) (WT(I), I=1, NOBS)
  501 FORMAT(5E11.4)
C
C
C         NPRMT IS THE NUMBER OF PARAMETERS TO BE IDENTIFIED
C         X IS A PARAMETER VECTORS
C         LIMIT IS A ITERATION LIMIT
C         IMED IS THE LOOP CRITERIA OF THE NEWTON-RAPHSON METHOD.
```

```
      NPRMT=8
      N=NSTATE
      LIMIT=7
      IMED=3
      OLDCOS=100.
      X(1)=0.0
      X(2)=0.0D0
      X(3)=0.0D0
      X(4)=0.0D0
      P(1)=0.19809D-01
      P(2)=0.233D-01
      P(3)=0.50133D-02
      P(4)=0.224669D-04
      P(5)=0.46385D-02
      P(6)=0.62262E5D-02
      P(7)=0.90691D-05
      P(8)=1.0D 00
C
C       PARAMETER IDENTIFICATION BY COMBINATION OF
C       NEWTON-RAPHSON, GAUSS-NEWTON, MODIFIED GAUSS-NEWTON METHOD.
C
      DO 90 ITRT=1, LIMIT
C
C       SUBROUTINE FUNCT(N, IHLF,1) COMPUTES THE GRADIENT VECTOR, G,
C       AND THE SENSITIVITY FUNCTION AND COST
C
   47 CALL FUNCT(N, IHLF, 1 )
      IF (IHLF .GE. 11) GO TO 92
C
      WRITE(6,601) OLDCOS, COST
  601 FORMAT(10X, 'OLDCOS=', E15.7/10X, '  COST=', E15.7)
C
C       CHECK WHETHER THE COST FUNCTION  IS LESS THAN
C       THE PREDETERMINED VALUE.
C
      IF (COST .LT. 1.0D-06) GO TO 92
```

```
C
C       CHECK WHETHER THE LOOP CRITERION EXCEEDED.
C
      IF (ITRT-IMED) 48, 48, 187
C
C       THE NEWTON-RAPHSON ITERATION METHOD.
C
   48 GSSUM=0.0D0
      DO 49 I=1, NPRMT
   49 GSSUM=GSSUM + G(I)**2
      DO 50 I=1, NPRMT
      RD(I)=COST*G(I)*(2.0D0)/GSSUM
   50 P(I)=P(I) + RD(I)
      WRITE(6,603)
      WRITE(6,605) (P(I), G(I), RD(I), I=1, NPRMT )
  603 FORMAT(10X, 'X(I)          G(I)          RD(I) ' )
  605 FORMAT(10X, 3E15.7)
C
C       CHECK WHETHER THE SOLUTION IS STABLE
C
  184 DO 185 I=1, NPRMT
      PCHCK=P(I)
      IF (PCHCK .GE. 0.0) GO TO 96
  185 CONTINUE
   85 DO 86 I=1, NSTATE
      RD(I)=RD(I)/2.0D0
   86 P(I)=P(I) - RD(I)
      WRITE (6,186)
  186 FORMAT( 10X, 'THE MAGNITUDE OF THE CHANGE VECTOR,P(I), ARE
     1 ', /10X, ' REDUCED BY SOME SCALE FACTOR' )
      WRITE(6,72) (P(I), I=1, NPRMT)
C
C       CHECK AGAIN WHETHER THE SOLUTIONS ARE STABLE
C
      GO TO 184
```

```
C
C       CHECK WHETHER THE NEW COST FUNCTION IS MINIMIZED.
C
   96 CST1=COST
      WRITE (6,72) (P(I), I=1, NPRMT)
      CALL FUNCT(N, IHLF, 2)
      CST2=COST
      ECST21=CST2-CST1
      IF (ECST21 .GT. 0.0D0) GO TO 85
      GO TO 90
C
C
C       MODIFIED GAUSS-NEWTON METHOD
C
C       MATRIX INVERSION OF  RA
  187 CALL DUMNV(RA, NPRMT, NPRMT, DTMT, L, M)
      WRITE (6,45) DTMT
      WRITE (6,46) ((RA(I,J),J=1,NSTATE),I=1,NSTATE)
   45 FORMAT(1H1, 10X, 'RA= INVERSE OF RA',//10X, 'DETERMINENT=',D15.7)
   46 FORMAT(  10X, 3E15.5)
C
C       THE MATRIX,RA SHOULD BE NON-SINGULAR
C
      IF (DTMT .LE. 0.1D-06) GO TO 92
      DO 70 I=1, NSTATE
      RC(I)=0.0
      DO 70 K=1, NSTATE
   70 RC(I)=RA(I,K)*G(K) + RC(I)
      WRITE(6,71)
      WRITE(6,72) (RC(I), I=1, NSTATE)
   71 FORMAT(//10X, 'RC = RA * RB ')
   72 FORMAT(10X, D25.14)
      WRITE (6,686)
  686 FORMAT(/10X, 'IDENTIFIED PARAMETERS' )
```

```
      DO 70 K=1, NSTATE
   70 RC(I)=RA(I,K)*G(K) + RC(I)
      WRITE(6,71)
      WRITE(6,72) (RC(I), I=1, NSTATE)
   71 FORMAT(//10X, 'RC = RA * RB ')
   72 FORMAT(10X, D25.14)
      WRITE (6,686)
  686 FORMAT(/10X, 'IDENTIFIED PARAMETERS' )
      DO 84 I=1, NSTATE
   80 P(I)=P(I) + RC(I)
      WRITE(6,83) P(I)
   83 FORMAT(10X, E15.7)
   84 CONTINUE
      GO TO 184
   90 CONTINUE
   92 STOP
      END
C
C
C
      SUBROUTINE FUNCT(N, IHLF, NC)
      IMPLICIT REAL*8(A-H, O-Z)
      COMMON X(12), G(08), FF(2, 8), RA(8,8), COST
      COMMON/CONST/DELT, NOBS, NPRMT, ITRT, LIMIT, NCST
      DIMENSION PRMT(5), VAR(32), DVAR(32), AUX(16,32)
      EXTERNAL FCT, OUPT
C
C
C
C       N IS NUMBER OF PARAMETERS
C       X IS PARAMETER VECTOR
C       G IS GRADIENT VECTOR
```

```
C
      IHLF=0
      NCST=NC
      NEQS=32
      MDLEQS=4
      IF (NCST .EQ. 2) NEQS=MDLEQS
      PRMT(1)=0.0
      PRMT(2)=(NOBS-1)*DELT
      PRMT(3)=2.0D0
      PRMT(4)=1.0E-1
      PRMT(5)=0.0
      DO 15 I=1, NPRMT
   15 G(I)=0.0
C
      PRMT4=1.0/NEQS
      DO 16 I=1, NEQS
      VAR(I)=0.0D0
   16 DVAR(I)=PRMT4
C
C
C
      CALL DHPCG(PRMT, VAR, DVAR, NEQS, IHLF, FCT, OUPT, AUX)
C
      WRITE(6,620) COST, IHLF
  620 FORMAT(10X, "COST=", E11.4, 10X, "IHLF=", I5 )
      RETURN
      END
C
C
C
```

```
      SUBROUTINE FCT(T, VAR, DVAR)
      IMPLICIT REAL*8(A-H, O-Z)
      COMMON X(12), G(90), FF(2, 8), RA(8,8), COST
      COMMON/INPUT/U1, U2, CV(1000), CF(1000)
      COMMON/CONST/DELT, NOBS, NPRMT, ITRT, LIMIT, NCST
      DIMENSION VAR(1), DVAR(1)
      REAL*4 CV, CF
C
      I=T/2.0 + 1
      U1=CV(I)
      U2=CF(I)
C
C       THE MODEL DYNAMICS
C
      DVAR(1)=-X(5)*VAR(1)-X(6)*VAR(2)+X(6)*VAR(3)
      DVAR(2)=-X(7)*VAR(2) + X(10)*U1
      DVAR(3)=VAR(4)
      DVAR(4)=-X(8)*VAR(3) - X(9)*VAR(4)+X(11)*U2
      IF (NCST .EQ. 2) GO TO 124
C
C       CALCULATION OF THE SENSITIVITY FUNCTION
C
      DO 17 J=5, 11
      DVAR(J)=-X(5)*VAR(J) -X(6)*VAR(J+7) + X(6)*VAR(J+14)
      DVAR(J+7)=-X(7)*VAR(J+7)
      DVAR(J+14)=VAR(J+21)
      DVAR(J+21)=-X(8)*VAR(J+14) - X(9)*VAR(J+21)
   17 CONTINUE
      DVAR(5)=DVAR(5) - VAR(1)
      DVAR(6)=DVAR(6) - VAR(2) +VAR(3)
      DVAR(14)=DVAR(14) -VAR(2)
      DVAR(17)=DVAR(17) + U1
      DVAR(29)=DVAR(29) - VAR(3)
      DVAR(30)=DVAR(30) - VAR(4)
      DVAR(32)=DVAR(32) + U2
  124 RETURN
      END
```

```
      SUBROUTINE OUPT(T, VAR, DVAR, IHLF, NEQS, PRMT)
      IMPLICIT REAL*8(A-H, O-Z)
      DIMENSION VAR(1), DVAR(1), PRMT(1)
      COMMON/CONST/DELT, NOBS, NPRMT, ITRT, LIMIT, NCST
      EXTERNAL FOBS
C
C       SAMPLE AT EACH GIVEN INTERVAL TIME, DELT
C
      IF (T .EQ. 0.0) ICOUNT=0
      TOBS=ICOUNT*DELT
      TCOMP=DABS(T-TOBS)
      IF(TCOMP-0.48828D-3) 102, 102, 104
  102 ICOUNT=ICOUNT + 1
  103 CALL FOBS(ICOUNT, T, VAR )
  104 RETURN
      END
C
C
C
      SUBROUTINE FOBS(M, T, VAR)
      IMPLICIT REAL*8(A-H, O-Z)
      COMMON X(12), G(08), FF(2, 8), RA(8,8), COST
      COMMON/OUTPUT/DT(200), WT(200)
      COMMON/CONST/DELT, NOBS, NPRMT, ITRT, LIMIT, NCST
      COMMON/INPUT/U1, U2, CV(1000), CF(1000)
      EQUIVALENCE(NSTATE, NPRMT)
      DIMENSION VAR(1)
      REAL*4 CV, CF
C
C       DEFINE OUTPUT DATA
C       DT IS THROTTLE PRESSURE
C       WT IS STEAM FLOW RATE
C
C       OBSERVATION
C
      ZWT=VAR(1)+X(12)*U1
      ZDT=VAR(3) - VAR(2)
```

```
C
C       DEFINE OBSERVATION ERRORS
C
      I=M
      OBSA=WT(I) -ZWT
      OBSB=DT(I) - ZDT
C
C       COMPUTE THE COST FUNCTION AT EACH ITERATIONS
C
      IF (T .EQ. 0.0) COST=0.0
      COST=OBSA**2 + OBSB**2 +COST
      WRITE (6,603) T, U1, U2, DT(I), ZDT, WT(I), ZWT
  603 FORMAT(10X, 7E12.4)
      IF (NCST .EQ. 2) GO TO 66
C
C
C       DEFINE SENSITIVITY FUNCTION
C
      DO 17 J=1, 7
      FF(1,J)=VAR(J+4)
   17 FF(2,J)=-VAR(J+11) + VAR(J+18)
      FF(1,8)=U1
      FF(2,8)=0.0D0
C
C       COMPUTE THE GRADIENT VECOR, G
C
      DO 63 N=1, NPRMT
      IF (T .EQ. 0.0) G(N)=0.0
      G(N)=FF(1,N)*OBSA +G(N) +FF(2,N)*OBSB
   63 CONTINUE
C
C
C
```

```
      DO 40 I=1, NSTATE
      DO 40 J=1, NSTATE
      IF (T .EQ. 0.0) RA(I,J)=0.0
   40 RA(I,J)=FF(1,I)*FF(1,J) + FF(2,I)*FF(2,J) + RA(I,J)
   66 RETURN
      END
```